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THREE-FIELD THEORY OF STRONG INTERACTIONS

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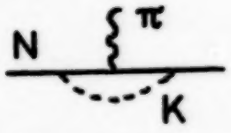
Abstract: A theory of strong interactions which uses only three massless fundamental fields is discussed. This theory gives all additive constants of the motion but not C and P separately. It is proved that the electric and pionic vertices have to conserve parity but not the vertices for β and strange particle interactions. It seems that the latter feature is not necessarily in contradiction with parity conservation in nuclear physics, in contrast with the opinion expressed in a previous paper on this question. Accepting the simplest representation for spin 0 bosons one finds that they are pseudoscalar rather than scalar inasmuch as parity is conserved.

1. Introduction

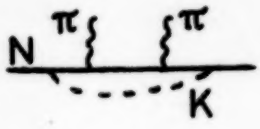
In a previous paper [1]) (quoted as I) it was investigated how many fundamental fields are necessary to obtain the quantities conserved in strong interactions. It turned out that in order to get the additive constants (baryon number N, isospin **T** and displacement of centre of charge U) and CP, only three massless Majorana (or Weyl) fields are needed. To get C and P separately one needs six fields, a possibility which could be realized by various kinds of fields. The more economical three-field case was discarded because of the high accuracy with which one knows that parity is conserved in nuclear physics. However, because of some recent developments this conclusion does not appear so sure any more. In the present paper we want to discuss this case in more detail.

Since CP is conserved in the three-field theory (TFT) the pion-nucleon vertex turns out to conserve parity. In fact, it was realized by several authors that CP-conservation and isobaric spin invariance exclude parity non-conserving Yukawa type (πN) couplings. However, parity may not be conserved in the NYK coupling, and the question arose whether also diagrams of the form



conserve parity. This turns out to be the case in the first few orders of perturbation theory and in I it was proved that in the TFT the exact π—N vertex

conserves parity for real nucleons. The proof does not apply if one of the nucleons is virtual and there is no argument why a diagram † of the form



should be parity conserving. Furthermore, there is no reason why the K-meson vertex should conserve parity, and hence there may be longitudinal polarizations in strange particle reactions: but this question is not yet settled experimentally. The parity non-conservation in π—N scattering due to strange particles has been estimated by Fubini and Walecka [2]). They used forward scattering dispersion relations. Since the vertex is parity-conserving the parity non-conservation does not enter via the inhomogeneous term but only via the reaction $\pi+\mathrm{N}\to\mathrm{Y}+\mathrm{K}$ which was assumed to exhibit maximum parity non-conservation (100 % longitudinal polarization of the Y). Using the experimental cross section for this reaction one finds even with such a drastic assumption that for low energy P—N scattering the amount of parity non-conservation is small. The mixing parameters between odd and even states ε_{ST} turn out to be (q denotes the momentum, ω the energy of the meson in the c.m.s.)

$$\varepsilon_{33} < 7\times 10^{-4}\frac{q}{\mu}\frac{\omega}{\mu}, \quad \varepsilon_{13} < 10^{-2}\frac{q}{\mu}$$

$$\varepsilon_{31} < 2\times 10^{-3}\frac{q}{\mu}\frac{\omega}{\mu}, \quad \varepsilon_{11} < 2\times 10^{-2}\frac{q}{\mu}.$$

Especially in the dominant 33 state there is almost no parity non-conservation. Therefore, the two-meson exchange potential will hardly either give any parity non-conserving nuclear force. Hence, mixing parameters $\approx 10^{-3}$ to 10^{-4} in nuclear physics may still be compatible with parity non-conservation for strange particles. Although this is only a semi-empirical consistency argument, we see at least that there is no immediate reason for discarding the TFT.

In the present paper we shall first develop the TFT using Weyl fields, which are a little more convenient than Majorana fields††. Next, we shall investigate the general structure of mesonic and electromagnetic vertices. It will turn out that the γ and π vertices but not necessarily the K-vertex, conserve parity. Furthermore, we shall study a threefield theory which is invariant under the full three-dimensional unitary group. In this theory, the K-vertex also conserves parity, but the multiplets contain some particles of which there does not seem to be any trace in nature. Thus, we do not find any reason for believing that a

† We use standard diagrams although the bosons are compound particles in the TFT.

†† The next two sections repeat, for the convenience of the reader, considerations already contained in I.

theory with a higher symmetry may provide a good first approximation. In this connection, we discuss the structure of bosons, in this theory, irrespective of the question of higher symmetries. It turns out that already the simplest possibility gives enough particles and that these bosons are pseudoscalar inasmuch as the vertex conserves parity. Needless to say, we can only answer questions of transformation properties and multiplicities but no dynamical ones, like stability and mass. Finally, we shall discuss the connection of the present approach to related work and point out which difficulties further progress has to overcome.

2. General Properties of Weyl Fields

A Weyl field is a complex two-component field ψ. A (proper) Lorentz transformation induces a unimodular transformation among the field components:

$$L\psi L^{-1} = M\psi, \quad |M| = 1, \quad L^{\dagger} = L^{-1}. \tag{1}$$

Since $\det(M) = e^{\mathrm{tr}(\ln M)}$, the most general unimodular 2×2 matrix can be written with Pauli matrices as

$$M = e^{\boldsymbol{\sigma}\cdot\mathbf{n}}, \tag{2}$$

$\mathbf{n}$ being an arbitrary complex vector. For $\mathbf{n}$ real, M corresponds to an "acceleration" in the direction of $\mathbf{n}$, for $\mathbf{n}$ imaginary to a rotation around the direction $\mathbf{n}$. Furthermore, one sees easily that

$$V_{\mu} = \psi^{\dagger}\sigma_{\mu}\psi, \quad \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3}$$

transforms like a four-vector, and that

$$\begin{aligned} s &= \psi g\psi + \psi^{\dagger} g\psi^{\dagger}, \\ Ps &= i(\psi g\psi - \psi^{\dagger} g\psi^{\dagger}), \end{aligned} \qquad g = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_2, \tag{4}$$

are invariants.

With these quantities one constructs the most general bilinear, first-order Lagrangian as

$$L_0 = i\psi^{\dagger}\sigma_{\mu}\psi^{\mu} + m\psi g\psi + \text{h.c.} \tag{5}$$

This Lagrangian is invariant under the proper Lorentz group, and also under two improper Lorentz-transformations: a unitary one

$$R\psi(\mathbf{x}, t)R^{-1} = g\psi^{\dagger}(-\mathbf{x}, t) \tag{6}$$

and an anti-unitary one

$$K\psi(\mathbf{x}, t)K^{-1} = g\psi(\mathbf{x}, -t). \tag{7}$$

Note that

$$K^2\psi K^{-2} = R^2\psi R^{-2} = -\psi. \tag{8}$$

The quantities (3) and (4) transform under the improper operations as follows:

$$\begin{aligned} RsR^{-1} &= s, \quad RPsR^{-1} = -Ps, \quad RV_\mu R^{-1} = -V^\mu; \\ KsK^{-1} &= s, \quad KPsK^{-1} = -Ps, \quad KV_\mu K^{-1} = V^\mu. \end{aligned} \tag{9}$$

Under both operations together the invariants are unchanged and the (axial) vector changes sign.

If $m = 0$, the Lagrangian L_0 becomes invariant under the transformation $\psi \to e^{i\alpha}\psi$. More generally, if L_0 is the sum of the Lagrangians for several massless Weyl fields,

$$L_0 = \sum_{a=1}^{n} \psi_a^\dagger \sigma^\mu \psi_{a,\mu} + \text{h.c.}, \tag{10}$$

it is invariant under unitary transformations of those fields among themselves:

$$U\psi_a U^{-1} = u_{ab}\psi_b, \quad u^\dagger u = 1, \quad UL_0U^{-1} = L_0. \tag{11}$$

If the canonical commutation rules are accepted, one can construct explicitly the infinitesimal generators for this transformation group:

$$U_{\text{inf}} = 1 + i \sum_{r=1}^{n} \delta\alpha^r \int \mathrm{d}^3x \psi_a^\dagger t_{ab}^{(r)} \psi_b, \tag{12}$$

where the t are a set of n^2 linearly independent hermitian $n \times n$ matrices. Since U commutes with the generators of the Lorentz group, the quantities

$$T^{(r)} = \int \mathrm{d}\sigma_\mu V_\mu^{(r)}, \quad V_\mu^{(r)} = \psi^\dagger \sigma_\mu t^{(r)} \psi \tag{13}$$

are constants and hence the currents $V^{(r)}$ are conserved:

$$V_\mu^{(r),\mu} = 0. \tag{14}$$

The T's are quantities of the nature of a charge and were used in I for defining the additive constants N, T, U of the strong interactions. To obtain the commutation properties of the T's with the improper operations it is convenient to take t's with definite reality properties:

$$t^{(a)\mathrm{T}} = -t^{(a)}, \quad t^{(s)\mathrm{T}} = t^{(s)}. \tag{15}$$

Then we find

$$\begin{aligned} RT^{(s)}R^{-1} &= -T^{(s)}, \quad RT^{(a)}R^{-1} = T^{(a)} \\ KT^{(s)}K^{-1} &= \quad T^{(s)}, \quad KT^{(a)}K^{-1} = -T^{(a)}. \end{aligned} \tag{16}$$

As expected all the T's change sign under RK, the latter corresponding to the usual TCP operation.

Of course, it may happen that the interaction † partly destroys the symme-

† We hesitate to write down an interaction explicitly since those non-renormalizable interactions are quite meaningless sofar. For the following we shall only assume that some interaction exists which shares some of the invariance properties of L_0.

tries and the total Lagrangian L is only invariant under a subgroup of the invariance group of L_0. In fact, the invariance group of the strong interactions (S.I.G.), which has as generators the baryon number N, the isobaric spin T and the displacement of centre of charge P, is isomorphic to the product of a 2-dimensional unitary unimodular group by two independent one-dimensional unitary groups †. In I it was noted that the smallest unitary group which contains the S.I.G. as a subgroup is the three-dimensional one. In this section we shall use only invariance under $\psi \to e^{i\alpha}\psi$, e.g. the transformation with $t = 1$. The corresponding T is N, the baryon number. Later on we shall use the full S.I.G. and also study an (unrealistic) theory which is invariant under the full three-dimensional unitary group.

Concluding this section we investigate the degeneracy of the eigenstates of the generators of the (inhomogeneous) Lorentzgroup. Since N commutes with them we can have common eigenstates of N with any one of them. It is important to note that the structure of the commutation relations between the various quantities implies that states with $P^2 \neq 0, N \neq 0$ are degenerate. States with $P^2 \neq 0$ can be transformed in such a way that $\mathbf{P} = 0$, and then we may build common eigenstates of P_0, $|\mathbf{J}|$, J_3 and N. Since R commutes with P_0, $|\mathbf{J}|$, J_3 and anticommutes with N, this state times R belongs to the same eigenvalues of the former operators and to the opposite eigenvalue of N. In particular, if there are discrete states with $P^2 \neq 0$, $N \neq 0$, those stable particles appear in doublets, the members belonging to opposite eigenvalues of N. Of course, whether a particular interaction will lead to this kind of mass spectrum, or whether the continuum will start at $P^2 = 0$, cannot be decided with the present mathematical techniques. Certainly, in perturbation theory those doublets will never appear as stable particles, since it is then assumed that incoming states with $P^2 = 0$ exist. Hence, there cannot be any stable massive particles with the same quantum numbers in a perturbation approach. However, there is no general argument why in a γ_5-invariant theory there should be no stable particles with non-vanishing mass.

3. Matrix Elements for one Field

We shall denote by $|p, \pm\rangle$ the eigenstates common to the displacement operator P_i and to N with eigenvalues $p_i \pm 1$:

† It is interesting to note that the S.I.G. is large enough so that any interaction invariant under it is automatically invariant under R and K. If N exists the interaction must contain equal numbers of $\psi^\dagger$ and ψ. Grouping them together the only Lorentz-covariant quantities are currents of the form (13). In general, there may be terms $V_\mu^{(a)} V^{\mu(s)}$ which are odd under R and spoil the invariance under improper transformations. However, the only quantities invariant under the S.I.G. are $V_\mu^{(N)}$, $V_\mu^{(V)}$ and $\Sigma_i V_\mu^{(T_i)} V^{\mu(T_i)}$ where the label in parenthesis indicates the constant constructed from these currents. All Lorentz-invariant combinations of these quantities are even under R. The K-invariance then follows from the T.C.P.-theorem. Within the framework of local field theory. we do not have to postulate invariance under the improper transformations, but get it as consequence of the other invariance requirements.

$$P_i|p, \pm\rangle = p_i|p, \pm\rangle, \quad N|p, \pm\rangle = \pm|p, \pm\rangle. \tag{17}$$

If P_0 is a purely timelike vector, we noted above that

$$R|p_0, \pm\rangle = \pm|p_0, \mp\rangle. \tag{18}$$

Of these states we are now interested in those which are discrete and have $|\mathbf{S}| = \frac{1}{2}$. They correspond to stable spin-$\frac{1}{2}$ particles and one would expect that there is some wave function which obeys the Dirac equation. In the present case this question is not completely trivial, since ψ is a two-component quantity, and furthermore the usual proof of the Dirac equation for certain matrix elements of four-component fields rests on the existence of a parity operation. We shall now show which is exactly the quantity which obeys a Dirac equation and how matrix elements can be expressed in terms of usual spinors.

Because of $[\psi, N] = \psi$, the state $\langle 0|N$ belongs to the eigenvalue 1 of N and we define a (two-component) wave-function U by

$$\langle 0|\psi(0)|p, +\rangle = U(p); \tag{19}$$

$\psi^\dagger$ gives only a matrix element to a $N = -1$ state:

$$\langle 0|\psi^\dagger(0)|p, -\rangle = V(p). \tag{20}$$

For the moment we consider only one ψ-field, in which case there will be 4 states for each p differing by the spin direction and the sign of N. The wave-function for the state with spin and $\mathbf{p}$ reversed can be obtained by time reversal $(p_t = p_0, -\mathbf{p})$:

$$\begin{aligned} &K|p, \mp\rangle = |p_t, \pm\rangle, \\ &U_t(p_t) = \langle 0|\psi(0)|p_t, \pm\rangle = gU^*(p). \end{aligned} \tag{21}$$

The transformation R reverses $\mathbf{p}$ and N; for timelike p_0 we find in particular

$$U(p_0) = gV(p_0). \tag{22}$$

However, the U for any p can be generated from $U(p_0)$ by a Lorentz transformation (see eq. (1)):

$$\begin{aligned} &|p, \pm\rangle = L|p_0, \pm\rangle, \\ &U(p) = MU(p_0), \quad V(p) = V(p_0)M^\dagger. \end{aligned} \tag{23}$$

Inserting this into (22) we get, by observing that $MM^\dagger = \sigma_\mu p^\mu/m$, and $m^2 = p^2$,

$$\begin{aligned} mgU &= \sigma_\mu{}^{\mathrm{T}} p^\mu V, \\ mgV &= \sigma_\mu p^\mu U. \end{aligned} \tag{24}$$

Introducing the 4-component spinor $\chi = \binom{U}{V}$ we see that eq. (24) is the Dirac equation

$$m\beta\chi = \alpha_\mu p^\mu \chi \tag{25}$$

in the representation

$$\beta = \sigma_2\rho_1, \quad \alpha_\mu = (1, \sigma_1, -\sigma_2\rho_3, \sigma_3), \tag{26}$$

with $\rho_1\binom{U}{V} = \binom{V}{U}$, etc. Under a proper Lorentz transformation χ transforms like a usual Dirac four-spinor and hence the physical interpretation of β and α-matrices is the usual one. In particular, the expression for the spin is $\chi^*\boldsymbol{\alpha}\times\boldsymbol{\alpha}\chi$. Assuming the asymptotic condition, the four states for a certain momentum can be generated by applying a limiting form of ψ multiplied by the two U^* or $\psi^\dagger$ multiplied by the two V^* unto the vacuum:

$$\begin{aligned} \langle p, +| &= \lim_{t\to-\infty} \langle 0| \int \mathrm{d}^3x \psi(x) U_p^*(x), \\ \langle p, -| &= \lim_{t\to-\infty} \langle 0| \int \mathrm{d}^3x \psi^\dagger(x) V_p^*(x). \end{aligned} \tag{27}$$

With the aid of these formulae we can express one-particle matrix elements of an operator O as follows:

$$\langle p'+|O|p+\rangle = U^*(p')\mathcal{O}(p', p)U(p), \tag{28}$$

where $\mathcal{O}(p', p)$ is a 2×2 matrix in spin space. If O is hermitian, $O^+ = O$, this implies

$$\mathcal{O}^+(p, p') = \mathcal{O}(p', p). \tag{29}$$

Furthermore, if O has a definite transformation property under time reversal, $KOK^{-1} = \pm O$, we find with (21)

$$g\mathcal{O}^*(p'_t, p_t)g = \pm\mathcal{O}(p, p'). \tag{30}$$

Let us first investigate the case when O is Lorentz-invariant: $\mathcal{O}$ must then be of the form

$$\mathcal{O} = a(p+p', \sigma)+ib(p-p', \sigma) \tag{31}$$

where for hermitian O eq. (29) tells us that a and b must be real functions of $(p-p)^2$. The expression (28) expressed in terms of the Dirac spinors then becomes

$$\langle p'+|O|p+\rangle = a\chi^*(p')\beta\chi(p)+b\chi^*(p')i\beta\gamma_5\chi(p), \tag{32}$$

i.e. a mixture of a scalar and a pseudoscalar matrix element. Furthermore, eq. (30) tells us that if O is invariant under time reversal, b must be zero and the matrix element is purely scalar, whereas if O changes sign under K, we have $a = 0$ and get a pseudoscalar vertex.

Next we investigate the case when O is a quantity V_μ which is hermitian, transforms like a vector and satisfies $KV_\mu K^{-1} = V_\mu$. Then we find that

$$\langle p'+|V_\mu|p+\rangle = U^*(p')\{a\sigma_\mu+b[p_\mu\sigma^\nu p'_\nu+p'_\mu\sigma^\nu p_\nu] \\ +ic[p_\mu\sigma^\nu p'_\nu-p'_\mu\sigma^\nu p_\nu]\}U(p) \tag{33}$$

with three real functions a, b, c. If V is also conserved, $V^\mu, \mu = 0$ i.e. $(p_\mu-p'_\mu)$ $O^\mu(p,p') = 0$, the vector $\mathcal{O}^\mu$ must be of the form

$$\mathcal{O}^\mu = a[\sigma^\mu(1-p^\nu p'_\nu)+p'^\mu\sigma^\nu p_\nu+p^\mu\sigma^\nu p'_\nu]+b(p^\mu+p'^\mu)\sigma^\nu(p_\mu+p'_\mu). \tag{34}$$

and the matrix element becomes, in terms of Dirac spinors,

$$\langle p'+|V_\mu|p+\rangle = 2a\chi^*(p')\alpha_\mu\chi(p)+2b(p_\mu+p'_\mu)\chi^*(p')\,\beta\chi(p). \tag{35}$$

Hence current conservation requires that the form of the current matrix element is the usual Dirac one + a Gordon current. The form (33) contains also a $\chi^*\alpha_\mu\gamma_5\chi$ term which cannot be excluded by time reversal. Thus, we see that in such a model these matrix elements for an invariant or a conserved current are reflection covariant. This is remarkable because in terms of bare (massless) particles the meaning of N is the difference between the number of right-screwed and left-screwed particles. The particles $|p,\pm\rangle$ are mixtures of massless spin-$\frac{1}{2}$ particles with definite helicity (of course, not all going in the same direction), the + set having a surplus right-screwed particle and the − set having a surplus left-screwed one. However, this intrinsic helicity does not show up as long as those particles interact with others via an invariant or a conserved current. That the particles $|+\rangle$ are not their own mirror image appears only when they interact through a current which is not conserved. From this point of view parity non-conservation in β-decay has nothing to do with the weakness of this interaction, but comes from the fact that the total current (including the axial vector part) need not be conserved. That electromagnetic and pionic interactions conserve parity stems from gauge invariance and the spin 0 of the pion.

However, there is another way how the intrinsic helicity of the fermions may come to the surface. Suppose there were two stable pairs of fermions $|\Lambda\pm\rangle$ and $|N\pm\rangle$. The arguments employed sofar do not allow us to conclude that a matrix element of the form $\langle+\Lambda|0|\mathrm{N}+\rangle$ is odd or even under reflection. This question will be studied in the next section where we return to the three-field case. Consequently there will, in general, be no parity conservation in such a theory. The virtual occurrence of those parity non-conserving vertices will influence other effects. For instance, the boson-fermion amplitude will be of the general form

$$T = \int \mathrm{d}^4x\langle p'|i[O(x), O(-x)]\mathcal{O}(x)\mathrm{e}^{iKx}|p\rangle = U^*(p')t(p', p, K)U(p)$$

and our usual arguments require

$$t^+(p, p', K) = t(p', p, -K) = -gt^*(p'_t, p_t, K_t)g.$$

Thus, t will contain terms of the form $ib\sigma_\mu K^\mu$. One readily finds $U^*(p)\sigma_\mu U(p) = p_\mu/m + s_\mu$ where the spin direction s_μ is orthogonal to p : $s_\mu p^\mu = 0$. Hence, even for forward scattering there will be a term $\sigma_\mu K^\mu$ which is not reflection invariant. However, it may be that under certain circumstances the parity non-conserving effects are small, as demonstrated in the work of Fubini and Walecka.

4. Matrix Elements for three Fields

The developments of the last section can be readily generalized to the three-field case. There it is convenient to consider ψ as a six-component quantity, transforming according to the sum of three unimodular representations under the Lorentz group. Furthermore, states of particles with the same mass will have a higher multiplicity according to the irreducible representations of the S.I.G. Accordingly, we use another label to characterize the one particle states and the wavefunction which now has six components:

$$\langle 0|\psi|Ap+\rangle = U_A(p).$$

The matrix elements will now be of the form

$$\langle +Bp'|O|Ap+\rangle = U_B^*(p')\mathcal{O}(p', p)U_A(p) \tag{37}$$

where $\mathcal{O}$ is a 6×6 matrix. If O is an invariant we get the same formula (31) where the hermiticity of O tells us that a and b are hermitian 3×3 matrices. However, time reversal now only tells us something about the symmetries of those matrices:

$$\begin{aligned} &\text{if } KOK^{-1} = 0: \quad a^{\mathrm{T}} = a, \quad b^{\mathrm{T}} = -b; \\ &\text{if } KOK^{-1} = -0: \quad a^{\mathrm{T}} = -a, \quad b^{\mathrm{T}} = b. \end{aligned} \tag{38}$$

Hence the vertex can be a mixture of a scalar and a pseudoscalar one. Some more restrictions may come from the covariance under an element U of the S.I.G. (11). Under U the wavefunction is transformed by a unitary 3×3 matrix

$$|A'\rangle = U|A\rangle, \quad U_{A'} = \langle 0|\psi U|A\rangle = uU_A. \tag{39}$$

Hence for matrix elements we have

$$\langle B'|O|A'\rangle = \langle B|U^{-1}OU|A\rangle = \langle B|O'|A\rangle = U_B^* u\mathcal{O}uU_A. \tag{40}$$

In particular, if O is the current of pions $\boldsymbol{\pi}$, which transforms under U like $\mathbf{T}$, it has to transform under U like $\left(\begin{array}{c|c}\boldsymbol{\tau} & 0\\ \hline 0 & 0\end{array}\right)$ and hence a and b must be proportional to this matrix. Thus we find † that if $K\pi_i K^{-1} = (-)^i\pi_i$ the vertex is purely

† Eqs. (28) and (29) of I have compensating mistakes in sign.

pseudoscalar, if $K\pi_i K^{-1} = (-)^{i+1}\pi_i$ it is scalar. Similarly, the matrix element of the electric current $\psi^\dagger\sigma_\mu(t_3+s)\psi$ must be proportional to $t_3+s = ({}^1{}_0{}_0)$ and the arguments of the last section apply. However, if B and A are particles belonging to different multiplets, e.g. Λ and P, there is no element in U which connects them and there is no further restriction of this kind. Hence the $\langle\Lambda|K|\mathrm{P}\rangle$ vertex can be parity non-conserving: U tells us only how this is related to $\langle\Lambda|K|\mathrm{N}\rangle$. Within our considerations there is no argument why parity should be conserved in strange particle interactions.

One might try to enlarge the symmetry group to get further restrictions. A possibility which naturally suggests itself is the full three-dimensional unitary group (T.U.G.) We shall now explore what multiplets of particles one should expect if this higher symmetry were present. The simplest possibility for bosons with $N = 0$ is that such states are already contained in the state

$$\psi_a^\dagger\psi_b|0\rangle.$$

These states do not yet form an irreducible representation of the T.U.G. since $\psi_a^\dagger\psi_a$ is an invariant subspace. It can be shown that it is the only invariant subspace so that those states contain a 1 and an 8-dimensional irreducible

TABLE 1

N	$\lvert T\rvert$	U	Dimension of repr.
0	0	0	1
0	$\frac{1}{2}$	-1 K^-	8
	0	0 π_0^0	
	1	0 π	
	$\frac{1}{2}$	1 K^+	
1	0	0	3
		occurs twice	
	$\frac{1}{2}$	1	
1	1	0	6
	$\frac{1}{2}$	1	
	0	2	
1	$\frac{3}{2}$	1 $(\pi^+ + \mathrm{P})$	15
	1	2 $(\mathrm{K}^+ + \mathrm{P})$	
	1	0 Σ	
	$\frac{1}{2}$	1 N	
	$\frac{1}{2}$	-1 Ξ	
	0	0 Λ	

representation of the T.U.G. †. Similarly we assume that all fermion states are contained in $\psi_a\psi_b^\dagger\psi_c|0\rangle$ and in table 1 we list the irreducible representations and the quantum numbers of U, N and T contained in them. Furthermore, in

† The following results are due to J. Wess and will be published in detail in a separate paper.

those representations which contain the observed particles we indicate their names. As far as the bosons are concerned the 8-dimensional representation contains all known bosons + the π_0^0. Within this higher symmetry all their masses should be degenerate whereas the other $T = 0$, $U = 0$ boson belongs to a different representation and may be unstable †. However, we now know that the π—K masses are quite different and if the mass of the π_0^0 were shifted by the same amount it would only show up as a resonance in the π—π scattering. Hence the apparent absence of the π_0^0 is no further argument against the T.U.G. For fermions we have a 15×15 representation which contains all known 8 fermions + two more systems which have the same quantum numbers as π^++P and K^++P. No such doubly charged particle has been established nor is any bound state or resonance of these systems known. Although the experimental evidence on such points may be incomplete, the T.U.G. does not seem to be a useful approximation. Whereas the Ξ—N mass splitting is considerable, the next $J = \frac{3}{2}$, $T = \frac{3}{2}$, $U = N = 1$ level appears at a much lower energy. Hence the splitting of the multiplets is larger than their separation, which makes this classification rather meaningless. Therefore, we reach the rather disappointing conclusion that the experimental material does not show any trace of this natural extension of the S.I.G., and hence there is no argument either why parity should be conserved in strange particle interactions.

Finally, we investigate the transformation properties of the bosons under the improper operations. We have seen that all observed bosons are already contained in $\psi^\dagger\psi|0\rangle$, which is a fortiori true if we restrict ourselves to the S.I.G. Roughly speaking, this means that the bosons consist, among other things, of a bare baryon+antibaryon. In the sense of the Fermi-Yang theory, we assume that this pair is in a relative s-state and the radial wavefunction has a short range. In a first approximation, we take both operators at the same point and form the quantity $\psi_a^\dagger(x)\sigma_\mu\psi_p(x)$. The bosons are to be spin-0 particles and characterized by a scalar field. Hence we put for the pion field $\pi^i(x)$

$$\pi^i_{,\mu}(x) = c\psi^\dagger(x)\sigma_\mu t_i\psi(x) + \ldots \tag{41}$$

where c is a normalization constant ††. From eq. (41) we conclude

$$K\pi^i K^{-1} = (-)^i\pi^i, \quad R\pi^i R^{-1} = (-)^i\pi^i. \tag{42}$$

Hence the pion field has exactly that transformation property under K which requires as pseudoscalar (rather than a scalar) vertex. Consequently, π has also the correct transformation property under R (which is the usual CP), which

† Those other particles have been mentioned by Okun [3]) in connection with the Sakata model. Within the S.I.G. one cannot make any statement about their masses; they may very well be unstable.

†† Since the term written out in (41) is divergence-free the pion would have mass zero; there must therefore be additional terms. One may speculate that the pion is the lightest strongly interacting field because (41) is already a pretty good approximation.

implies, for instance, that the meson field of the antinucleon has the opposite sign to that of the nucleon.

5. Conclusions

We have seen that three Weyl fields are enough to give all conserved quantities for strong interactions except for parity. However, some special interactions, as the pionic and electromagnetic ones, may conserve parity. There is no general reason within such a model that parity should be conserved in strange particle interactions, and experimental data bearing on this point should be watched closely. Of course, all our analysis only demonstrates that in the TFT the construction of all known strongly interacting particles as compound structures does not meet with any contradiction from the point of view of group theory. How such a construction could be dynamically effected is a far more difficult question.

An attempt at a theory of elementary particles has been reported by Heisenberg and his collaborators [2] †. Apart from dynamical questions which we did not touch, their approach differs from the present one inasmuch as it uses only two Weyl fields. According to the analysis of I such a theory may contain N and T but not U. Hence the hope that in this theory strange particles will emerge does not seem to have any foundation. Furthermore, the lepton number is supposed to emerge from scale transformations. Also this assumption seems arbitrary, since in usual theories the generator for scale transformations has a continuous eigenvalue spectrum. Furthermore, it is not clear how the reduction of the internal symmetry which is implied by electric and weak interactions comes about. The same question arises in the work of Gürsey [5], which agrees with the present one as far as the counting of the fields is concerned. However, he also wants to get leptons and needs, therefore, one more field. Another difficulty one meets by including leptons is that some features like the absence of $\mu^{\pm} \to e^{\pm} + e^{+} + e^{-}$ seem to come from peculiarities of the interaction (no neutral currents). These requirements are not contained in the conservation laws and it is not easy to see how they can come out of a compound model.

We conclude with some general remarks on compound particles. Usually one tries to write down a specific Hamiltonian, which means that one tries to express the generators of the Lorentz group in terms of the local fields. However, one knows that in relativistic theories such expressions contain infinite renormalization constants, and there is no good example of bound states in such theories. But one should remember that there is actually no need for expressing the generators in terms of the local fields. They can be easily expressed

† I am grateful to Prof. Heisenberg for obtaining a preprint prior to publication.

in terms of the asymptotic fields and it is misguided ambition to express the generators or their commutators with local quantities in terms of the local fields. What one really wants is a causal expression of the local fields in terms of the asymptotic fields. A theory with compound particles is one in which the number of asymptotic fields exceeds the number of local fields. The problem is now to prove that this smaller number of local fields already forms a complete set. This is the point where the difficulty is hidden. On the one hand, the canonical completeness, which requires that the local fields at a certain time are a complete set, leads to infinite renormalization constants. On the other, taking all fields within a time interval as complete is far too weak a condition and does not exclude trivial examples in which one scalar field describes several kinds of particle. It seems that mathematics has to be developed further before we can answer the question of atomic theory: what is the number of fundamental constituents of matter?

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THE DISINTEGRATION OF La^{131}

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Abstract: The decay of La^{131} has been studied with the help of a magnetic lens spectrometer, scintillation spectrometers and scintillation coincidence spectrometers. The half-life is 61 ± 2 min. Three positon groups have been found having end-point energies and relative abundances as follows: 1.939 ± 0.045 MeV (27 %), 1.424 ± 0.036 MeV (56 %), 0.704 ± 0.045 MeV (17 %). Gamma rays are found at 115, 169, 214, 254, 285, 364, 417, 445, 511 (annihilation radiation), 597 and 878 keV. The line at 115 keV is strongly internally converted. A disintegration scheme is proposed.

1. Introduction

The disintegration scheme of La^{131} is essentially unknown. The only work reported in the literature is that of Gransden and Boyle [1]) who bombarded barium with protons of energies up to 90 MeV and found two new lanthanum activities. The mass numbers of these activities were determined with the help of a mass spectrometer. An activity of 58 min half-life, attributed to La^{131}, gave rise to positons and gamma rays. Absorption measurements yielded an energy of 1.6 MeV for the positons. From the beta-ray systematics, Way and Wood [2]) have determined the total disintegration energy to be ≈ 3 MeV.

The present work was undertaken to make a more detailed study of the radiations from La^{131}.

2. Source Preparation

An electromagnetically separated sample, enriched in Ba^{130}, was bombarded by deuterons in the Indiana University cyclotron. The isotopic constitution of the source is given in table 1. The lanthanum activities were separated, carrier

TABLE 1
Isotopic constitution of Ba source

Mass number	130	132	134	135	136	137	138
Atomic percent	15.4	0.8	5.4	9.8	8.6	9.7	50.3

† Supported by the Joint Program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

free, by a procedure described [3]) in recent work on La^{135}. Sources were prepared for measurement of gamma rays on a scintillation spectrometer, beta rays in a magnetic lens spectrometer and beta-gamma coincidences using scintillation equipment.

3. Measurements

The barium was bombarded for one hour with 11.5 MeV deuterons. Such short irradiation times suppress any long-lived isotopes formed by the bombardment of Ba^{134}, Ba^{136}, Ba^{137} and Ba^{138}. Owing to the short half-lives of La^{134} and La^{136}, and the length of time necessary to perform the chemistry, any possible activity from these substances is not seen.

3.1. HALF-LIVE DETERMINATION

A measurement of the half-life of La^{131} was made with the help of several types of experiments. The total activity of a typical separated source was followed with a Geiger counter. In this experiment, the half-lives seen were a long one (20 hours or greater) which was very weak, one of 4 hours and one of 63.5 minutes. Using the scintillation equipment (see below), the half-life associated with the strong gamma rays was determined. With the exception of the X-ray, these decayed with a half-life of 62 ± 3 min. In measuring the positon spectrum in a magnetic lens spectrometer, several positon energies were taken as check points and the half-lives at these energies determined. These gave a value of 61 ± 2 min. In view of these figures, we estimate the half-life to be 61 ± 2 min.

3.2. GAMMA-RAY SPECTRUM

The energies and intensities of the gamma rays were measured on a scintillation spectrometer consisting of a $3'' \times 3''$ cylindrical NaI(Tl) crystal, a 5" photomultiplier and 100-channel analyzer. The scintillation spectrometer was calibrated as to energy using various standard sources and also with respect to intensities with the help of the tables of Wolicki, Jastrow and Brooks [4]) and the curves of Heath [5]). Several experiments were carried out, of which a typical example is shown in fig. 1. The energies and relative intensities of these gamma rays are shown in table 2.

The errors placed on the energies were the average deviations as determined from five experiments. Those of the relative intensities were the average deviations determined from two runs.

In making the measurements listed in table 2, the source was covered on each side with enough copper to stop all positons, so that all intensities can be given in terms of the number of 511-keV quanta. The intensities have been corrected for the absorption in copper. There appear to be no gamma rays of

any appreciable intensity at energies greater than 878 keV. There is a long high-energy tail which is assumed to arise from annihilation of positons in flight. This background is weak and falls off logarithmically as the energy

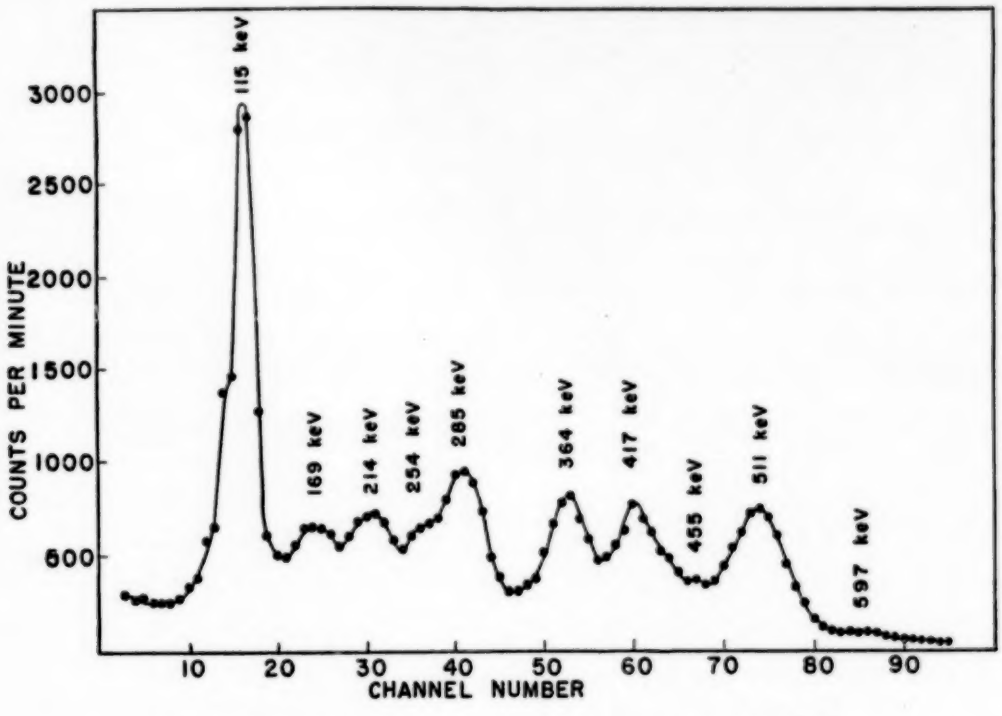


Fig. 1. Scintillation spectrum of the gamma rays of La^{131}.

TABLE 2

Energies and relative intensities of gamma rays of La^{131}

Energy (keV)	Relative Intensity
115±2	0.82±0.05
169±2	0.19±0.05
214±3	0.28±0.02
254±2	0.28±0.02
285±3	0.61±0.01
364±2	0.72±0.03
417±2	0.71±0.05
455±8	0.29±0.06
511	2.00
597±8	0.25±0.02
878±2	0.04±0.01

increases. Very weak gamma rays of energies approximately 1 MeV and 1.2 MeV have been observed, but these have a considerably longer half-life and are assumed to arise from a weak lanthanum activity which is not La^{131}.

The intensity of the X-ray was compared to that of the line at 115 keV, without using any copper radiator on the source. The decays of the X-ray and of the 115-keV line were followed with the help of the 100-channel analyzer. The X-ray decayed with a composite half-life composed of a 17.5-hour part and a 61-minute part. The relative intensities of the X-ray and 115-keV line were determined in the early part of the run when 83 % of the X-ray intensity was owing to La^{131}. After correction for efficiency, escape, and absorption, the relative intensity of the X-ray to the 115-keV line was 3.1. Using the same basis of comparison that is given in table 2, the two intensities are: X-ray, 2.54 ± 0.3; 115 keV, 0.82 ± 0.05.

In order to determine the number of electron captures, the X-ray intensity has to be corrected for a contribution arising from the internal conversion of various lines of which the 115-keV line is the strongest, for the fluorescent yield and for the contribution of L capture. Since all gamma rays and the X-ray are measured in terms of annihilation radiation and the number of K and L+M electrons of the 115-keV line and the number of K electrons for the other weakly converted lines have been measured with respect to the total number of positons, it is easy to correct the X-ray intensity and that of the 115-keV line for the influence of internal conversion. Thus, the number of vacancies in the K shell arising from electron capture and internal conversion is

$$\frac{(I_X/N_{\beta^+})}{w_K} = \frac{2.54}{0.875} = 2.90\pm0.3.$$

Here w_K is the fluorescent yield and N_{β^+} is obtained from the intensity of the annihilation radiation. The number of K vancacies arising from internal conversion of the 115-keV line is

$$\frac{\text{No. of K vacancies for 115-keV line}}{N_{\beta^+}} = \frac{\text{K+L+M}}{\text{Total } \beta^+ \text{ spectrum}} \times \frac{\text{K}}{\text{K+L+M}}.$$

From the section on the particle spectrum,

$$\frac{\text{K+L+M}}{\text{Total } \beta^+ \text{ spectrum}} = 0.62\pm0.03; \quad \frac{\text{K}}{\text{K+L+M}} = \frac{6.3}{7.3}.$$

Hence, the number of K vacancies arising from internal conversion in the 115-keV line is 0.54 ± 0.02. The contribution to the K vacancies from the other internally converted lines is 0.05, making a total of 0.59 ± 0.02. Hence, the number of K vacancies from electron capture is 2.31 ± 0.32. Using Zweifel's [6]) value of the ratio L/K = 0.122 (L electron capture to K electron capture) for $Z = 56$, the total electron capture is 2.59 ± 0.35 with the total positon emission as 1.00. Thus, electron capture comprises 72 % and positon emission 28 % of the disintegrations.

3.3. THE PARTICLE SPECTRUM

The particle spectrum was measured in a magnetic lens spectrometer without a baffle to separate positons from negatons. Owing to the short half-life of

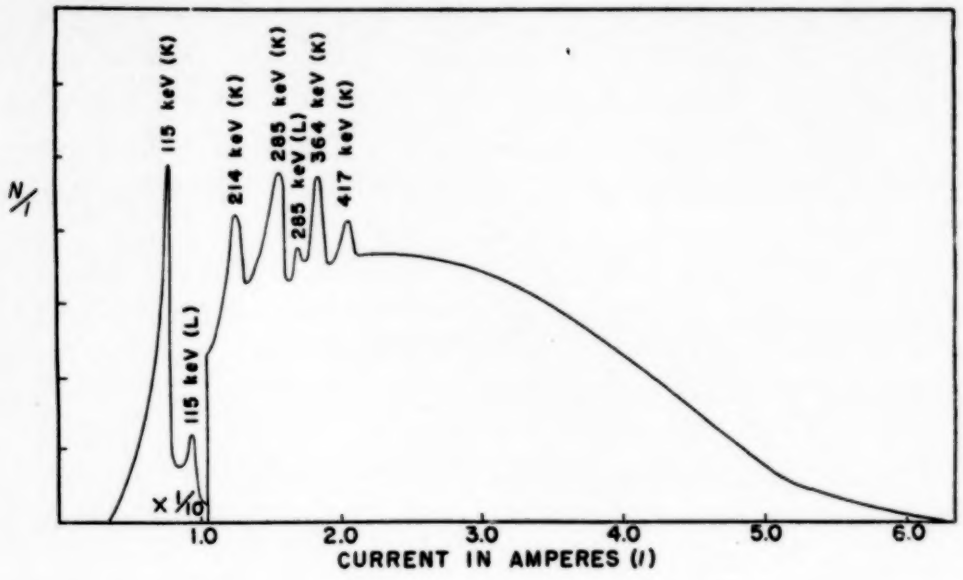


Fig. 2. Particle spectrum of La^{131}.

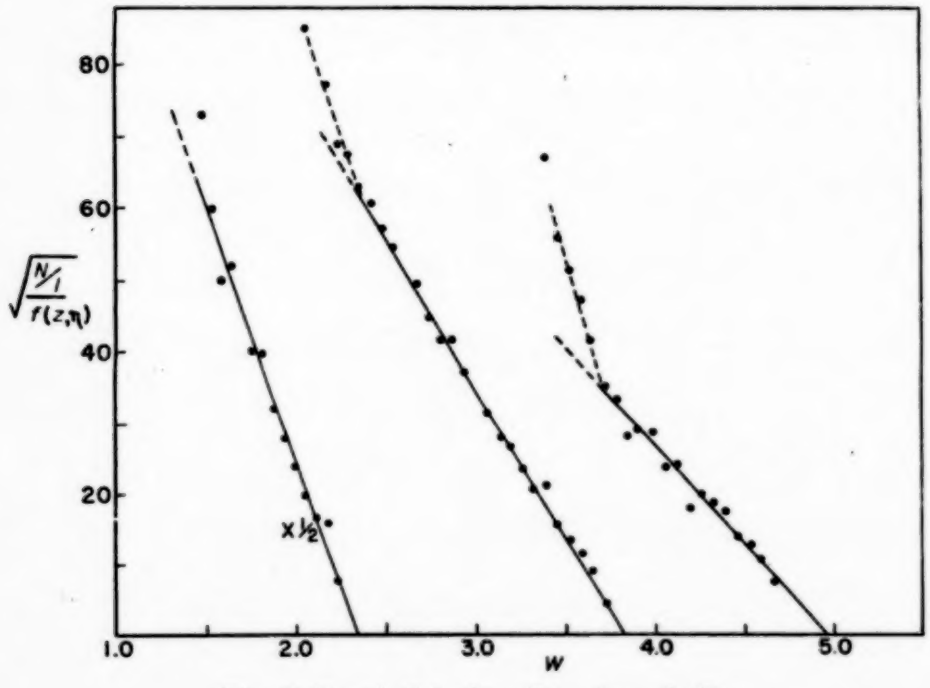


Fig. 3. Fermi plot of positons from La^{131}.

the material, many experiments had to be performed and a composite made of the results. A typical distribution is shown in fig. 2. In addition to the positon distribution, several internal conversion lines are seen. The strongest lines are

the K and L lines for the 115-keV gamma ray and the weaker lines correspond to gamma rays as noted in fig. 2.

A Fermi plot was made of the positon distribution from each of five complete experiments. A typical example is shown in fig. 3. Three positon groups were found whose end-point energies and relative abundances are given in table 3.

TABLE 3

Distribution of positon groups from La^{131}

End-point energy (MeV)	Relative abundance (Percent)	Relative abundance per disintegration	log ft
1.939 ± 0.045	27 ± 8	0.075 ± 0.023	5.9
1.424 ± 0.036	56 ± 11	0.157 ± 0.03	5.2
0.704 ± 0.045	17 ± 3	0.047 ± 0.009	5.4
Total	100	0.279	

In column 3 of the table is given the relative abundance per disintegration. The last column gives log ft ($f = f_+ + f_K$), obtained with the help of the tables of Feenberg and Trigg [7]).

The value of the K internal conversion coefficient for the line at 115 keV can be obtained from the relative intensities of the K and L+M lines to that of the total positon spectrum. The results are

$$K/N_{\beta^+} = 0.54 \pm 0.02; \quad K/(L+M) = 6.3 \pm 0.8.$$

The internal conversion coefficient is given by

$$\alpha_K = \frac{K}{N_{\beta^+}} \frac{1}{I_{115}} = \frac{0.54 \pm 0.02}{0.82 \pm 0.05} = 0.66 \pm 0.06.$$

Using the tables of Sliv and Band [8]), the nearest values of the theoretical coefficients to that observed for this energy and Z are $\alpha_2 = 0.78$; $\beta_1 = 0.56$. The observed value of α_K lies approximately midway between the values for M1 and E2 radiations. The theoretical values of $K/(L_I+L_{II}+L_{III})$ are: E2, 2.6; M1, 7.4. The latter value favors the M1 over the E2 assignment.

3.4. COINCIDENCE EXPERIMENTS

Various gamma-gamma and positon-gamma coincidence experiments were performed. In the gamma-gamma coincidence experiments, two $1\frac{1}{2}'' \times 1\frac{1}{2}''$ NaI(Tl) crystals were used as detectors, and were connected to a fast-slow coincidence circuit in which the coincidences were displayed on a 100-channel analyzer. One branch contained a single-channel pulse height analyzer which

could be set on various desired gamma-ray peaks. Coincidences observed when the single-channel analyzer is set on the peaks at 878, 597, 364, 285 and 115 keV are shown in figs. 4a to 4e.

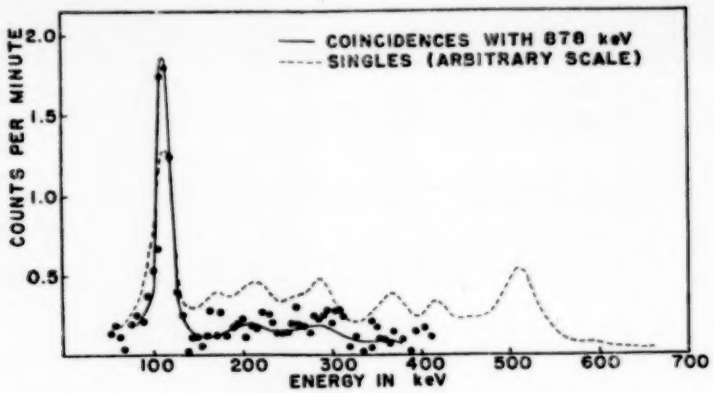
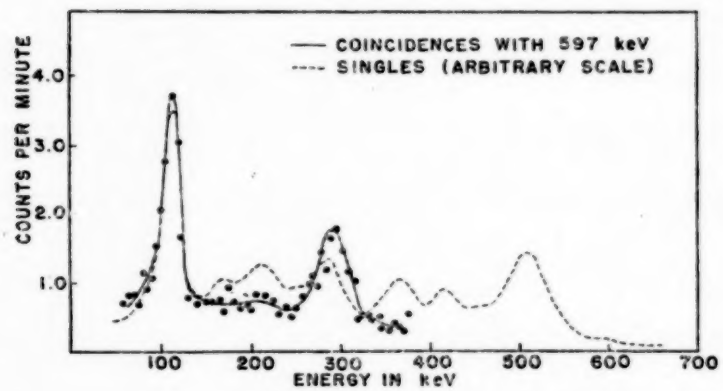
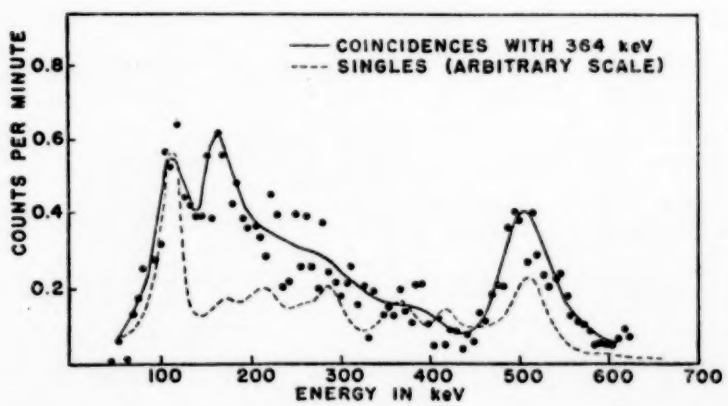


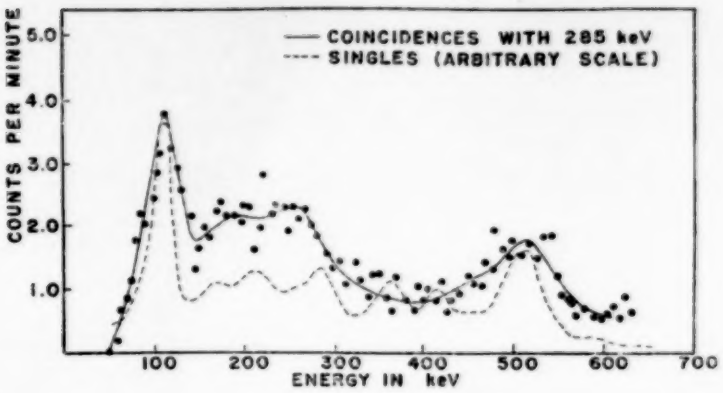
Fig. 4. Gamma-gamma coincidences in La^{131}.
4a. Coincidences with 878 keV line.



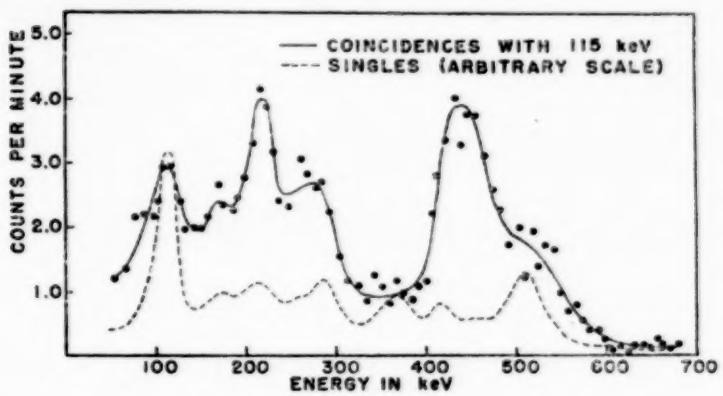
4b. Coincidences with 595 keV line.



4c. Coincidences with 364 keV line.



4d. Coincidences with 285 keV line.



4e. Coincidences with 115 keV line.

The highest energy line, that at 878 keV, is definitely in coincidence with the line at 115 keV. The line at 597 keV, which is relatively weak, is strongly in coincidence with a line at 285 keV. The coincidences with the line at 115 keV probably arise from the presence of radiation due to annihilation in flight. The line at 364 keV is in coincidence with annihilation radiation and a line at 169 keV, but *not* with the line at 285 keV. For the lower energy lines, corrections were not made for the contribution of Compton-scattered gamma rays riding under the photo-peaks. In the case of coincidences with the line at 285 keV, it is significant that there are no strong coincidences with the line at 364 keV. The line at 115 keV shows strong coincidences with the complex at 417—455 keV and a line at 214 keV and no strong coincidences with that at 364 keV.

In order to measure positon-gamma coincidences, one of the NaI(Tl) crystals was replaced by an anthracene crystal. This was calibrated as to energy with the help of several internal conversion lines. The method used was to select a given region of the positon distribution with an anthracene counter and to display those gamma rays in coincidence with this energy region on the

100-channel analyzer. If the positon channel accepts a region between 1.56 and 1.64 MeV, beyond the end-point of the second group, the coincidence spectrum shows no marked gamma-ray peaks. Even the line at 115 keV appears to be absent. On the other hand, when the positon channel accepts an energy region between 1.09 and 1.17 MeV, within the second group, strong coincidences are seen at 115, 169, 214, 364 and 511 keV.

4. Discussion of Results

From all of the information obtained in the foregoing, it is possible to assign tentatively a disintegration scheme for La^{131}. From the positon distribution alone, it is not possible to say whether the most energetic group goes to the

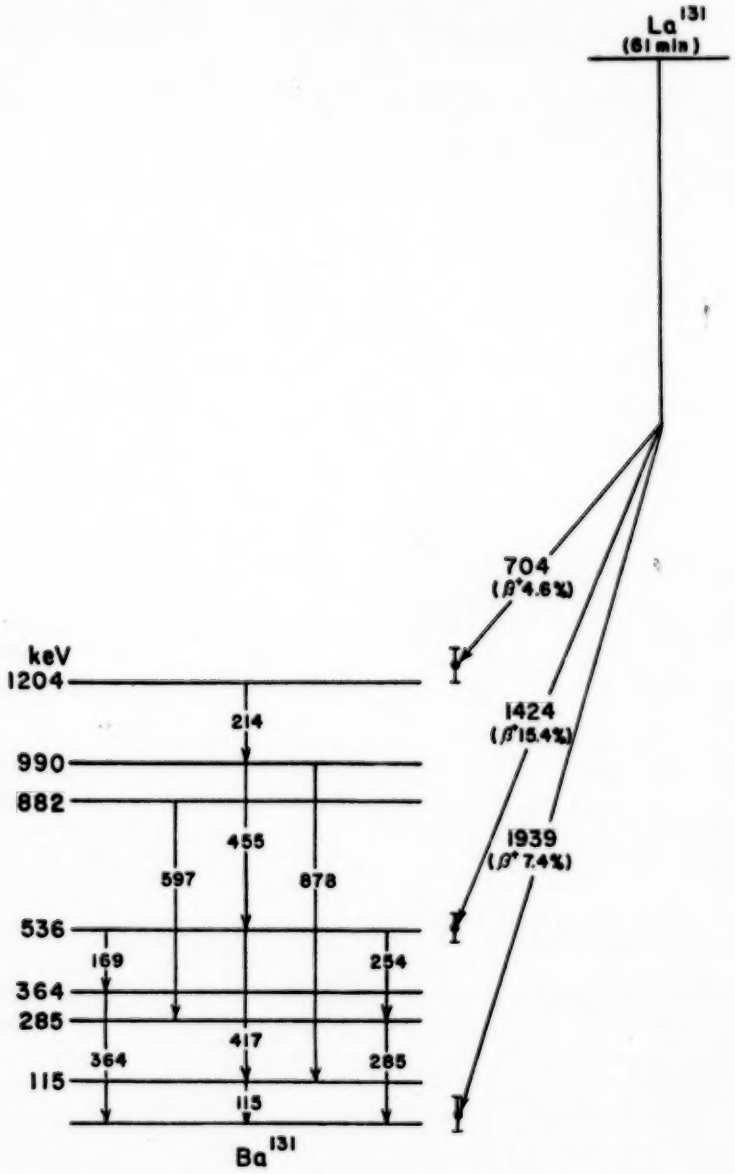


Fig. 5. Tentative disintegration scheme of La^{131}.

ground state or a state at 115 keV above the ground state. However, the positon-gamma coincidence experiments definitely show that the high energy group is not in coincidence with the gamma ray at 115 keV. This would then give a total disintegration energy of 2.96 MeV, in agreement with the prediction of Way and Wood [2]). From the energies, relative intensities of the gamma rays and from the gamma-gamma coincidence experiments, the scheme shown in fig. 5 is proposed. Within the limits of error of measurement, positons populate the ground level and the states at 536 keV and 1204 keV. Using the proposed scheme, the intensities of the gamma rays as given in table 2 and the internal conversion data for the various conversion lines, the probability of populating the various levels has been computed and is shown in table 4.

TABLE 4

Values of probability of electron capture plus positon emission to various levels

Level (keV)	Electron capture plus positon emission (percent)
1204	0.082
990	0.010
882	0.070
536	0.249
364	0.150
285	0.027
115	0.190
0	0.222

The isomeric level ($h_{\frac{11}{2}}$) which one would expect in Ba^{131} was not seen in the La^{131} decay. This was looked for but not found in the barium fraction, owing to the production of other barium isotopes, mostly Ba^{139} and Ba^{135}. This situation is similar to that found in La^{135} in which [3]) the $h_{\frac{11}{2}}$ level of Ba^{135} is not populated by the decay of La^{135}.

It is not possible to say very much about the spins and parities of the levels of Ba^{131}. Arguing by analogy with the case of La^{135}, it is supposed that the ground state of La^{131} has the configuration $d_{\frac{3}{2}}$. Since the highest energy group goes to the ground state rather than to the first excited state, one would infer that the ground state of Ba^{131} has the configuration $d_{\frac{3}{2}}$ and the first excited state $s_{\frac{1}{2}}$. This is essentially in agreement with the internal conversion data.

The authors are indebted to Professor M. B. Sampson and the cyclotron group for making the bombardments and to Mr. Louis Ross for making the chemical separations. They are also indebted to Mr. T. D. Nainan for help in taking some of the data.

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LEVELS IN ^{200}Hg POPULATED FROM THE DECAY OF ^{200}Au

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Abstract: The decay of 48 minute ^{200}Au was investigated. Only three gamma rays (370, 1225 and 1600 keV) were found to follow the decay of this isotope. Excited states at 370 and 1595 keV in ^{200}Hg can accommodate these gamma rays. No gamma rays were found to correspond to the excitation or the deexcitation of a level at 950 keV, which was observed in both the neutron capture gamma ray work on ^{199}Hg and in the beta decay investigations of ^{200}Tl. Evidence is presented for a new gold activity with a half-life of 19 ± 2 hours.

1. Introduction

The level scheme of ^{200}Hg has been studied [1,2]) from the decay of ^{200}Tl and also from the gamma rays following thermal neutron capture [3,4]) in ^{199}Hg. A number of levels has been established as a result of these studies. The first 2^+ level is known [5,6]) to lie at 0.37 MeV. The presence of this level was confirmed by studies performed by various groups [7]). A second 2^+ level has not, so far, been established with certainty among the many known excited states. The absence of a cross-over transition from the second excited level at 950 keV, together with the fact that the stop-over transition deexciting this level was found to be a pure E2 transition, led Herrlander and Gerholm [1]) to assign spin and parity 4^+ to this level. Lindqvist [8]) performed angular correlation measurements for the cascades involving the level at 950 keV. Although no definite spin assignment could be made for this level from these measurements, the last author suggested a spin 4^+ for the 950 keV level as the most plausible one.

In the neutron capture gamma ray measurements an $\approx$ 950 keV gamma ray was observed [3,4]). From gamma-gamma coincidence measurements, Segel [4]) placed this gamma ray as a ground state transition from the second excited state in ^{200}Hg, thus assigning a spin and parity 2^+ to this level.

Recently, Roy and Roy [9]) studied the decay of 48 min ^{200}Au which also leads to levels in ^{200}Hg. Only gamma rays of 367, 1230 and 1600 keV were reported. The last gamma ray was proposed to be the ground state transition from the level of the same energy, in agreement with the results of neutron capture gamma ray work [3,4]). This transition was not reported by Herrlander

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et al. [1]). Gupta and Jha [2]) reported a peak at 1.58 MeV (uncertain) in the decay of ^{200}Tl. No gamma rays corresponding to the deexcitation of the 950 keV level were reported to follow the decay of ^{200}Au.

In order to search for gamma rays which can correspond to the deexcitation of a second 2^+ level and to confirm the presence of a ground state transition from the 1600 keV level, the decay of ^{200}Au was studied.

2. Source Preparation and Chemical Procedure

Sources of ^{200}Au were produced by a (d, α) reaction on mercury. Targets of mercury sulfide were irradiated with the 25 MeV deuteron beam of the synchrocyclotron of this Institute for $\approx 5\,\mu$Ah. The targets were dissolved in HNO_3 and HCl. Carriers of Tl, Hg and Au were added. The solution was diluted and Au was reduced with Na_2SO_3. The solution was then filtered and the precipitate was washed with a solution of $TlNO_3$.

The Au fraction was dissolved in HNO_3 and HCl, holdback carrier of Tl was added and the Au was precipitated again. This last process was repeated four times and the precipitate was then dried and heated in order to evaporate the mercury; only the Au fraction should stay behind.

This chemical procedure took $\approx$ 50 minutes and the sources were examined directly after the end of the separation.

The activity was also produced by fast neutron bombardment of thallium; these sources were too weak for accurate intensity determinations.

3. Measurements and Results

3.1. DIRECT GAMMA RAY SPECTRUM

Two detectors were used to examine this spectrum. The first consisted of a cylindrical NaI(Tl) crystal of 25 mm diameter and 25 mm height coupled to a 6292 DuMont photomultiplier. The second was a cylindrical NaI(Tl) crystal of 62 mm $\times$ 62 mm size coupled to a 6363 DuMont photomultiplier. This last crystal was mainly used to examine the energy region between $\approx$ 1000 and 2000 keV. Both crystals were calibrated for photopeak detection efficiency in the regions of interest. The spectra were displayed on a 100 channel analyzer (RIDL).

The decay of the sources was followed for $\approx$ 24 hours. The half-life of the ^{200}Au peaks was 48 ± 1 minute in agreement with the recent determination by Roy *et al.* [9]) The contributions of the long-lived Au activities present in the source were subtracted. These contributions did not result in an important distortion of the spectrum at the start of the measurement.

The gamma ray spectrum of ^{200}Au is shown in fig. 1 in an energy region from $\approx$ 200 to 1500 keV. The energy values and intensities of the various peaks analyzed in the spectrum are listed in table 1.

No other gamma rays were found to belong to the decay of ^{200}Au with certainty. The energy of the 1225 keV peak was determined several times with special care in order to decide whether this gamma ray corresponds to the de-excitation of either the 1595 keV or the 1574 keV level known [1]) to be populated from the decay of ^{200}Tl.

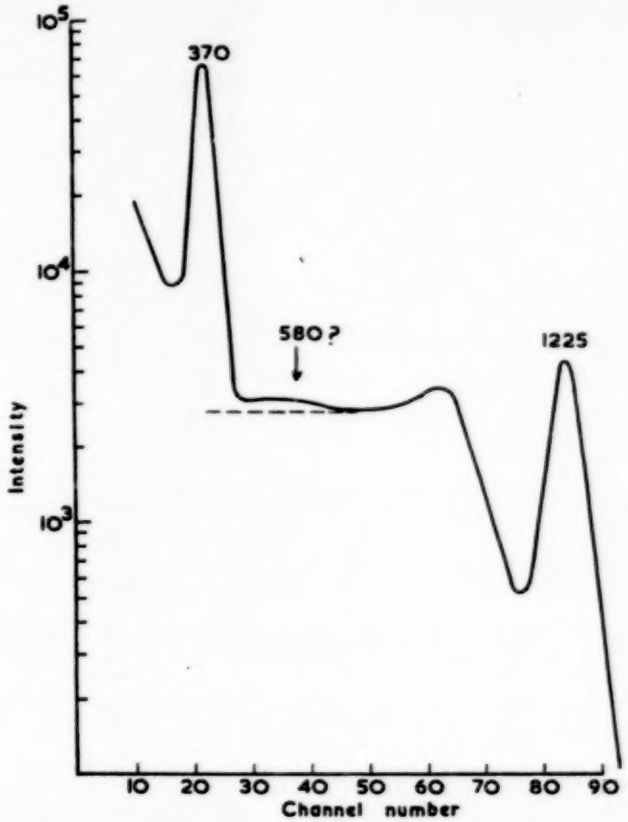

Fig. 1. Gamma ray spectrum of ^{200}Au in an energy region from ≈ 200 to 1500 keV (measured using the 25 mm × 25 mm crystal).

TABLE 1

Energies and relative intensities of gamma rays following the decay of ^{200}Au, normalized to 100 for the 370 keV gamma ray

Roy [9]) scin		present work scin	
Energy (keV)	Intensity	Energy (keV)	Intensity
367 ± 5	100	370 ± 5	100
1230 ± 10	100 ± 5	1225 ± 5	80 ± 8
1600 ± 20	< 5	1600 ± 10	2 ± 1
		No 580 keV γ	< 1.0
		No 650 keV γ	< 1.0
		No 950 keV γ	< 3.0

In order to make sure whether the 1600 keV peak is a real gamma ray or entirely due to summing effects, sources were measured with the 62×62 mm crystal through 10 mm of lead ($\approx$ 4 half thicknesses for the 370 keV gamma ray). The lead absorbers were sandwiched between 3 mm copper sheets to absorb the fluorescent K X-ray of lead. A source of ^{22}Na was examined in the same geometry using the same absorbers to estimate the amount of residual summing in the 1600 keV peak under these conditions. In this manner it was shown that the 1600 keV peak is not entirely due to summing; the intensity of the real 1600 keV peak applying various corrections for crystal detecting efficiency and absorption is reported in table 1. This intensity value is not in disagreement with the corresponding value quoted by Roy *et al.* [9]) who estimated it to be less than 5 % of the 1225 keV peak.

The main disagreement between the results reported in this work and those of Roy *et al.* [9]) occurs in the intensity value of the 1225 keV peak. The last authors report that the 1225 and 370 keV gamma rays are of equal intensities. The value quoted in table 1 was checked several times. The shape of the 1225 keV peak was obtained from the 1277 keV peak in ^{22}Na. The exact fit between the two peaks indicates that the 1225 keV peak is single. The shape of the 370 keV peak was interpolated from the 411 keV peak from ^{198}Au and the 325 keV peak from ^{51}Cr; this peak also corresponds to a single gamma ray.

3.2. COINCIDENCE RELATIONS

The larger crystal described in 3.1. has a well along its axis which has a diameter of 9 mm and a depth of 31 mm; it is provided with a copper tube of

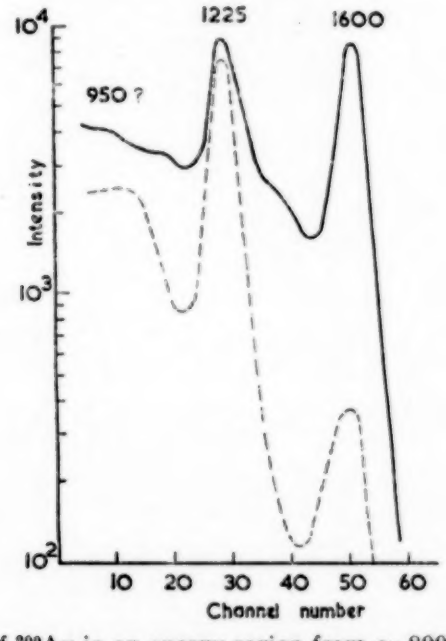

Fig. 2. Summing spectrum of ^{200}Au in an energy region from $\approx$ 800 to 2000 keV. The dashed line represents the direct spectrum in the same energy region with the source placed 5 cm from the well-type crystal; a part of the 1600 keV peak is due to summing.

1 mm thickness to stop negatons of energy up to $\approx$ 4 MeV. This crystal was used to examine the summing spectrum of ^{200}Au.

Fig. 2 shows this summing spectrum in the energy region between $\approx$ 800 and 2000 keV, with the source placed inside the well. The dashed line represents the spectrum in the same energy region with the source placed 5 cm from the crystal. The sharp rise in the height of the 1600 keV peak confirms the coincidence between the 1225 and 370 keV gamma rays. No important summing at 950 keV was observed. No summing peaks with energy $>$ 1600 were found.

A further check on the coincidences was made with the slow coincidence ($2\tau = 1\mu$s) facilities available in the RIDL kicksorter. It was shown, by selecting the 0.37 MeV transition as the gating peak with a single channel analyzer, that the 1.23 MeV peak is in coincidence with it, but that neither the 1.59 MeV, nor a second 0.37 MeV transition is present in the coincident spectrum.

3.3. THE BETA RAY SPECTRUM

The negaton spectrum of ^{200}Au was examined using a stilbene crystal, 1 cm thick and 4 cm in diameter, mounted on a 6292 DuMont photomultiplier. The crystal was calibrated for energy with ^{90}Sr+^{90}Y (2.25 MeV), ^{32}P (1.71 MeV) and ^{207}Bi (916 keV conversion peak). The end point of this spectrum was found to lie at 2.20$\pm$0.10 MeV, in agreement with the value obtained by Roy *et al.* [9]). The Fermi-Kurie plot of this spectrum shows a slight deviation from linearity at $\approx$ 400 keV below the end point, which may indicate the presence of a group of negatons of energy $\approx$ 1.8 MeV. The intensity of such a branch cannot be given with reasonable accuracy since a slight change in the intensity of the higher energy branch would result in a very large change in the intensity of the relatively much weaker 1.8 MeV group. The intensity of the expected branch with 650 keV maximum energy cannot be estimated, owing to the importance of the back-scattering of the 2.20 MeV group in this region of energy.

The decay of the negatons was followed; it yielded the same value for the half-life of ^{200}Au as mentioned above.

4. Discussion

A decay scheme for ^{200}Au is shown in fig. 3. The value (4.0$\pm$0.4) quoted by Roy *et al.* [9]) for the ratio between the intensity of the negatons to that of the 1225 plus the 1595 keV gamma rays was adopted. The intensities of the various negaton branches feeding the different levels in ^{200}Hg can then be calculated from the gamma intensities; they are given on the decay scheme.

Roy *et al.* [9]) suggested a spin and parity of 0^- for the ground state of ^{200}Au, in view of the absence of a negaton branch to the first 2^+ level. However, the present work makes the assignment of a spin and parity of 1^- to this ground

state equally possible, since the log *ft*-values for the negaton transitions to the ground state (7.0 ± 0.1) and the first excited state (7.9 ± 0.2) are compatible with normal first forbidden transitions (log $f^1t = 8.2\pm0.2$ for this last branch).

The existence of a ground state transition from the 1595 keV level seems certain. This transition has an intensity of less than 3 % of that of the 1225 keV transition; it has not been observed with certainty [2]) in the spectrum of ^{200}Tl, which is not surprising, since it would only have an intensity of ≈ 0.3 % per disintegration.

The log *ft*-value of 5.5 ± 0.1 for the negaton group from ^{200}Au feeding the 1595 level suggests an allowed transition. A spin of 0, 1 or 2 with negative parity should then be assigned to this level. A spin of 0 is ruled out since a ground state gamma transition from the 1595 keV level was seen. A spin of 2^-

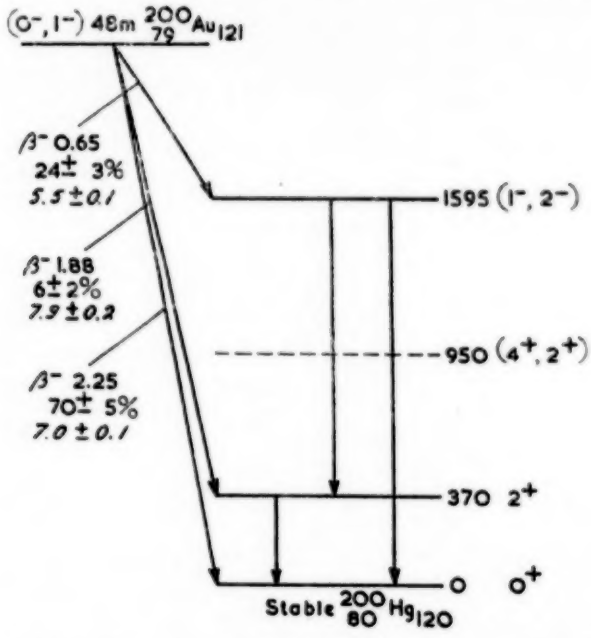

Fig. 3. Decay scheme of ^{200}Au. The energy and intensity (in percentage per disintegration) of the various beta branches following the decay of ^{200}Au are shown on the decay scheme. The log *ft*-values for these branches are also given in italics.

seems to be preferable, since otherwise the ground state transition would probably have been much stronger; this argument is not very decisive, however, since E1 transitions are known to be slowed down by large factors in some cases.

A suitable candidate for the second 2^+ level in ^{200}Hg would be the 950 keV level. Its energy is 2.57 times that of the first 2^+ level, corresponding well with the value 2.63 in ^{198}Hg and 2.19 in ^{202}Hg. However, the absence of gamma rays corresponding to the deexcitation of such a level (see table 1) may indicate that this level has a spin > 2, since otherwise it should have been populated by a negaton transition from the decay of ^{200}Au (unless this is a first forbidden unique branch) and by a gamma transition from the 1595 keV level. The last

transition should have been detectable (according to single particle calculations [10])). It follows, that a level of energy $\approx$ 950 keV in ^{200}Hg probably does not have a spin 2, in disagreement with the suggestions made by Segel [4]). The level of that energy in ^{200}Hg populated from the decay of ^{200}Tl then, most probably, has a spin of 4^+ in agreement with suggestions made by Herrlander *et al.* [1]) and by Lindqvist [8]).

5. Evidence for a New Gold Activity

In all sources produced by deuteron bombardment of mercury a residual activity was observed after the decay of the 48 minute period. To study it in more detail, stronger sources were produced (30 μAh) which were left to decay for $\approx$ 6 hours before the chemical separation was started. In order to ascertain the assignment to gold, repeated separations from mercury, and especially from thallium, were performed.

The gamma ray spectrum of the new activity showed a definite peak at 740 ± 20 keV, and possible ones at 255 ± 30, 500 ± 20 and 880 ± 30 keV, decaying with a half-life of 19 ± 2 hours. Identical results were obtained from at least five different bombardments. In two cases the mercury and thallium fractions obtained from the same target were also studied with the same gamma ray spectrometer: these spectra were completely different in structure. Especially the spectrum of ^{200}Tl was studied in view of the closeness of the value of its period (26 hours) to that of the new activity; again it was clear that the two were not identical.

Gold isotopes with mass numbers between 193 and 204 can be produced from 25 MeV deuteron bombardment of natural mercury. The gold fraction clearly showed the presence of the isotopes with mass numbers 196, 198, 199 and 200. In view of the low abundance of some of the mercury isotopes one does not expect observable quantities of gold isotopes with mass numbers 193, 194, 203 and 204 to be present.

In the course of other studies on gold activities performed at this Institute, platinum was bombarded with deuterons, both in the natural composition and enriched in mass 196. In none of the gold sources produced in this manner a $\approx$ 19 hour period was detected with an appreciable intensity. Neither was it found in sources produced by fast neutron bombardment of gold.

From this it would follow that the mass number of the new gold activity is probably higher than 197.

The presence of impurities which follow the gold fraction in the chemical separation can, of course, not be excluded completely. However, a search through the literature revealed no nucleide with a half-life between 16 and 22 hours with a gamma ray of 740 keV, except ^{97}Zr and ^{95}Tc. The presence of the first can be excluded since no strong 660 keV gamma ray was present, that of

the second cannot be ruled out on analogous grounds, but it seems highly unlikely.

On the basis of these considerations the new 19 ± 2 hour activity is tentatively assigned to a gold isotope with $A \geqq 198$.

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LEVELS IN ^{200}Hg STUDIED FROM THE DECAY OF ^{200}Tl

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Abstract: The gamma ray spectrum from the decay of ^{200}Tl has been investigated with a scintillation spectrometer. The ground state transitions from the 1595, 1732 and 1885 keV levels in ^{200}Hg have been detected. From summing studies and from the direct gamma ray spectrum the existence of new levels at 2000 ± 20, 2280 ± 20 and 2400 ± 30 keV in ^{200}Hg, populated from the decay of ^{200}Tl, is inferred.

1. Introduction

The decay of ^{200}Tl has been investigated in detail by Herrlander and Gerholm[1]) and excited states of ^{200}Hg have been placed at 368, 947, 1575, 1995, 1660, 1732, 1776, 1885 and 2137 keV. The reaction ^{199}Hg(n, γ)^{200}Hg has been investigated by many groups and as a result of this a level scheme of ^{200}Hg has been adopted by the Nuclear Data Group [2]), mainly on the basis of the results of Segel [6]). Although a considerable amount of agreement exists between the level scheme of ^{200}Hg proposed from the decay of ^{200}Tl and that from the neutron capture gamma ray work, some discrepancies exist. The ground state transitions from the 1595 and 1732 keV levels have been observed in the capture gamma ray work [2]), but these transitions are not reported in ref. [1]). Although gamma rays from the decay of ^{200}Tl have been observed in this region [3]), their intensities and energies were not determined precisely. A recent investigation on the decay of ^{200}Au, which also leads to the excited states of ^{200}Hg, proved the existence of a ground state transition from the 1595 keV level [4, 5]). As will be discussed below, the branching ratio from this level, as observed by different groups, does not seem to be in agreement. Another discrepancy exists with regard to the cross-over transition from the 947 keV level. An upper limit of 0.05 % is placed on the K-conversion line of the 947 keV transition relative to that of the 368 keV transition from the study [1]) of ^{200}Tl and an assignment of 4^+ was therefore preferred for the 947 keV level. In the neutron capture gamma ray studies, Segel [6]) finds the intensity of the 947 keV gamma ray to be approximately equal to that of the 579 keV gamma ray and an assignment of 2^+ was therefore preferred for the 947 keV level. The angular

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correlation studies of the 368 and the 579 keV gamma rays in the decay of ^{200}Tl were not decisive for the spin assignment to the 947 keV state, although the authors preferred an assignment of 4^+ for this level [7]).

It was mainly to investigate more carefully the higher energy cross-over gamma rays and the 947 keV cross-over gamma ray from the decay of ^{200}Tl, that this work was undertaken.

2. Source Preparation

The ^{200}Tl activity was produced by bombarding mercuric sulfide with $\approx$ 25 MeV deuterons in the synchrocyclotron of this Institute. The irradiated target was left to cool off for two days, after which it was dissolved in aqua regia with 50 μgm of Tl carrier; gold and platinum holdback carriers were also added. The solution was made 1 *N* with hydrobromic acid. Nickel powder was added to reduce the gold. The thallium activity was extracted by ether saturated with 1 *N* hydrobromic acid.

3. Direct Gamma Ray Spectrum

The low energy gamma ray spectrum in the region from 300 to 1600 keV was measured with a cylindrical NaI(Tl) crystal (diameter 25 mm, height 25 mm), coupled to a 6292 DuMont photomultiplier. In order to avoid summing the source was kept at a distance of $\approx$ 15 cm from the crystal. The high energy gamma ray spectrum (in the region from 800 keV to 2800 keV) was measured with a well-type NaI(Tl) crystal (diameter 62 mm, height 62 mm) and a distance of $\approx$ 11 cm was kept between the source and the crystal. The spectra were displayed on a 100 channel RIDL analyzer. The half-life of the entire spectrum was checked relative to that of the (1207+1227) keV photopeak for about three days: no contribution from other activities was indicated. Owing to the low intensity of the high energy part of the gamma ray spectrum, long runs (up to 10 hours) were necessary; the contribution due to the background was also measured for the same time and subtracted. The spectrum in the high energy region is shown in fig. 1. The contribution of internal bremsstrahlung was calculated according to the theory of Glauber and Martin [8]) on the basis of the decay scheme given in ref. [1]). This contribution amounts to $\approx$ 8 % at the highest energy gamma ray peak (2280 keV), while at the 1207 keV peak it is only $\approx$ 0.1 %. The gamma ray spectrum was analyzed by successively peeling off the pulse height distributions due to the various component gamma rays starting at the high energy end. The gamma ray spectra from sources of ^{22}Na, ^{42}K, ^{88}Y, ^{65}Zn, ^{54}Mn, ^{137}Cs, ^{51}Cr, ^{85}Sr, ^{144}Ce and ThC were used for energy calibration and to construct the line shapes of the different peaks resolved in the gamma ray spectrum by interpolation. Table 1 lists the various gamma

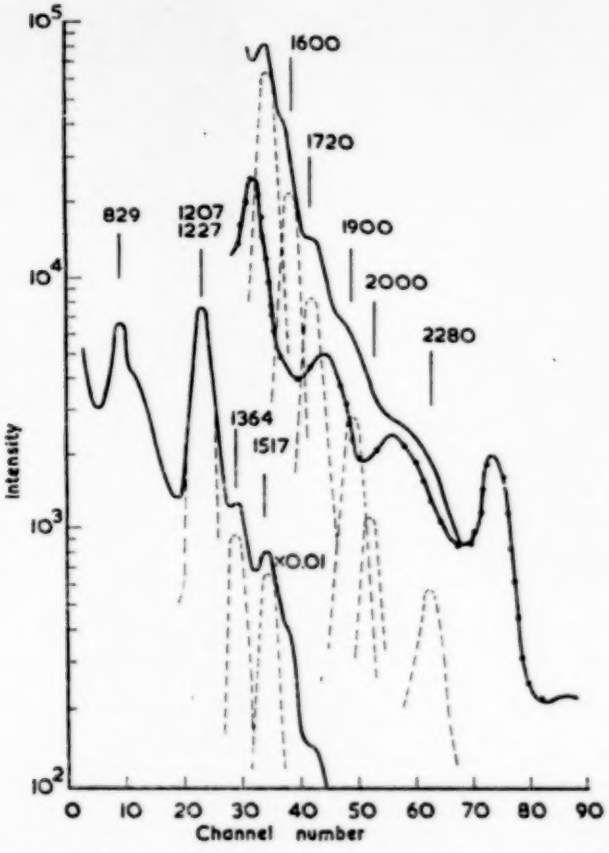

Fig. 1. Direct gamma ray spectrum of ^{200}Tl in an energy region from ≈ 800 to 2800 keV; the line with dots represents the background contribution and the dashed peaks represent the resolved photopeaks after subtraction of the contribution due to background and inner bremsstrahlung. The energies of the photopeaks are indicated in keV.

TABLE 1

Energies and relative intensities of the gamma rays following the decay of ^{200}Tl

Gamma ray energy (keV)	Rel. int. Herrlander [1])	Present work	$\alpha_K \times 10^2$	Theoretical $\alpha_K \times 10^2$					
				E1	E2	E3	M1	M2	M3
368	100	100	—	—	(3.9)	—	—	—	—
579	14	18 ±2	1.37±0.20	0.55	1.4	3.6	5.5	14	32
(680)	—	≤6	—	—	—	—	—	—	—
787	—	3.1 ±0.4	1.05±0.30	0.30	0.77	1.8	2.4	6.0	12.5
829	7	11.4 ±1.2	2.2 ±0.3	0.27	0.70	1.55	2.05	5.0	10
(947)	<0.7	<1	—	—	—	—	—	—	—
1207+1227	30	35.0 ±3.5	—	—	—	—	—	—	—
1364	—	5.3 ±0.6	0.73±0.18	0.12	0.28	0.57	0.56	1.40	2.6
1517	≈2	4.5 ±0.5	0.43±0.10	0.093	0.24	0.44	0.45	1.02	1.8
1600± 20	—	0.32±0.06	—	—	—	—	—	—	—
1720± 20	—	0.39±0.08	—	—	—	—	—	—	—
1900± 20	<0.1	0.12±0.03	≦1.0	0.067	0.15	0.29	0.26	0.57	1.04
2000± 20	—	0.08±0.02	—	—	—	—	—	—	—
2280± 20	—	0.04±0.01	—	—	—	—	—	—	—

rays resolved in this manner. The relative intensities are normalized to a value of 100 for the 368 keV gamma ray. The contribution due to summing for the high energy gamma rays, which was not completely negligible in the geometry used with the larger crystal, was obtained experimentally by studying the ^{22}Na spectrum in the same geometry and applying a correction for the relative photo-efficiency of the 368 keV gamma ray compared with the 511 keV gamma ray. Table 1 also gives the intensities obtained by Herrlander *et al.* [1]). Gamma rays of 629 and 712 keV [1]) could not be resolved from the spectrum. The K-conversion coefficients α_K are calculated using the K-conversion line intensities listed in ref. [1]) and by normalizing to the theoretical value of α_K (0.039) for the pure E2 368 keV transition [9]); these are also given in table 1. The table also allows comparison with the theoretical K-conversion coefficients [9]) for M1, M2, M3, E1, E2 and E3 multipolarities.

4. Summing Spectrum

Additional information regarding the higher energy levels can be derived from the summing spectrum. This summing spectrum was examined by placing the source inside the well of the well-type crystal (diameter of the well 9 mm, depth 31 mm). The well was provided with a copper tube of 2 mm thickness to cut down the intense KX-ray. The summing spectrum from ^{200}Tl in the energy region of 800 to 2800 keV is shown in fig. 2. A comparison of this summing spectrum with the direct gamma ray spectrum shows that the intensities of the (1227+1207) and 829 keV photo peaks are reduced and that there are sum peaks at 950 and 1580 keV and broad summing bumps at 1800$\pm$100 keV and 2200$\pm$100 keV. A close comparison shows that the summing peak at 1580$\pm$20 keV is 20 keV lower than the 1600$\pm$20 keV peak resolved in the direct spectrum, indicating that the level at 1575 keV in ^{200}Tl is fed more than the level at 1595 keV from the electron capture decay of ^{200}Tl and that the 1595 keV level emits a gamma ray to the ground state while the 1575 keV level probably does not decay by a ground state transition. The summing bump at 1800 keV must be composed of at least three peaks of 1660, 1780 and 1880 keV. The summing spectrum in the region from 2000 to 2400 keV shows a weak sum peak at 2400$\pm$30 keV, a peak at 2280$\pm$20 keV almost entirely due to single gamma rays, a sum peak at 2140 keV falling in the valley of the 2280 keV gamma ray and a sum peak at 2000 keV. The highest energy sum peak at 2400$\pm$30 keV has already been reported before [3]); it must correspond to a level in ^{200}Hg at the same energy. This level has been identified in the neutron capture gamma ray work [2]) as decaying via a cascade of 2020 and 370 keV transitions. From the present work it is difficult to get information regarding its decay modes. From the comparison of the intensity of the 2400 and 2000 keV peaks in the summing spectrum it can be inferred that the 2000 keV gamma ray,

observed in the direct spectrum, cannot originate totally at the 2400 keV level. This, together with the fact that the intensity of the 2000 keV peak is larger in the summing spectrum compared with the direct gamma ray spectrum, indicates a level at 2000±20 keV which must have additional decay modes apart from the direct ground state transition. Thus this work gives an indication for three more levels in ^{200}Hg populated in the decay of ^{200}Tl than already postulated by Herrlander *et al.* [1]), viz. at 2400±30, 2280±20 and 2000±20 keV.

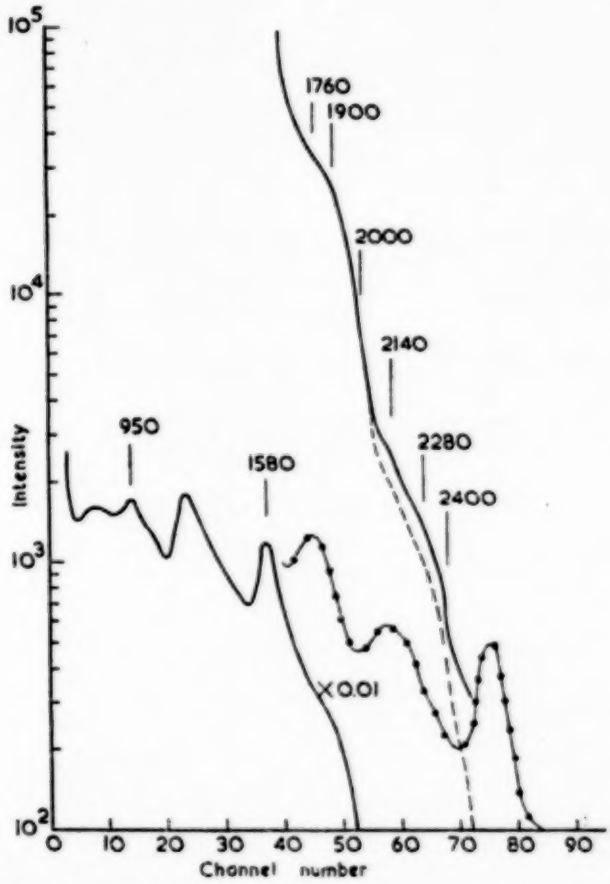

Fig. 2. Summing spectrum of ^{200}Tl in an energy region from ≈ 800 to 2800 keV; the line with dots represents the background contribution, the dashed line represents the summing spectrum due to ^{200}Tl. The energies of the peaks are indicated in keV.

5. Discussion

The results of the measurements reported in the present work and those of Herrlander *et al.* [1]) are reasonably in agreement. A level scheme for ^{200}Hg has been constructed on the basis of these combined results. A discussion follows below.

The 0^+ and 2^+ character for the ground state and first excited state (368 keV) of the even nucleus ^{200}Hg is rather certain [2]). Different assignments have been proposed for the second excited state at 947 keV. From the present work, the

intensity of a possible 947 keV gamma ray is less than 1 % of that of the 368 keV gamma ray and an assignment of 4^+, in agreement with Herrlander *et al.* [1]) and Lindqvist [7]), is indicated for this level. The origin of the discrepancy between the present work and that by Herrlander *et al.* [1]) on the one hand and that by Segel [6]) on the other hand, cannot be resolved as yet. As pointed out by Segel [6]), the assumption of a close doublet of levels at ≈ 950 keV may partially explain the discrepancy, but this leads to other difficulties.

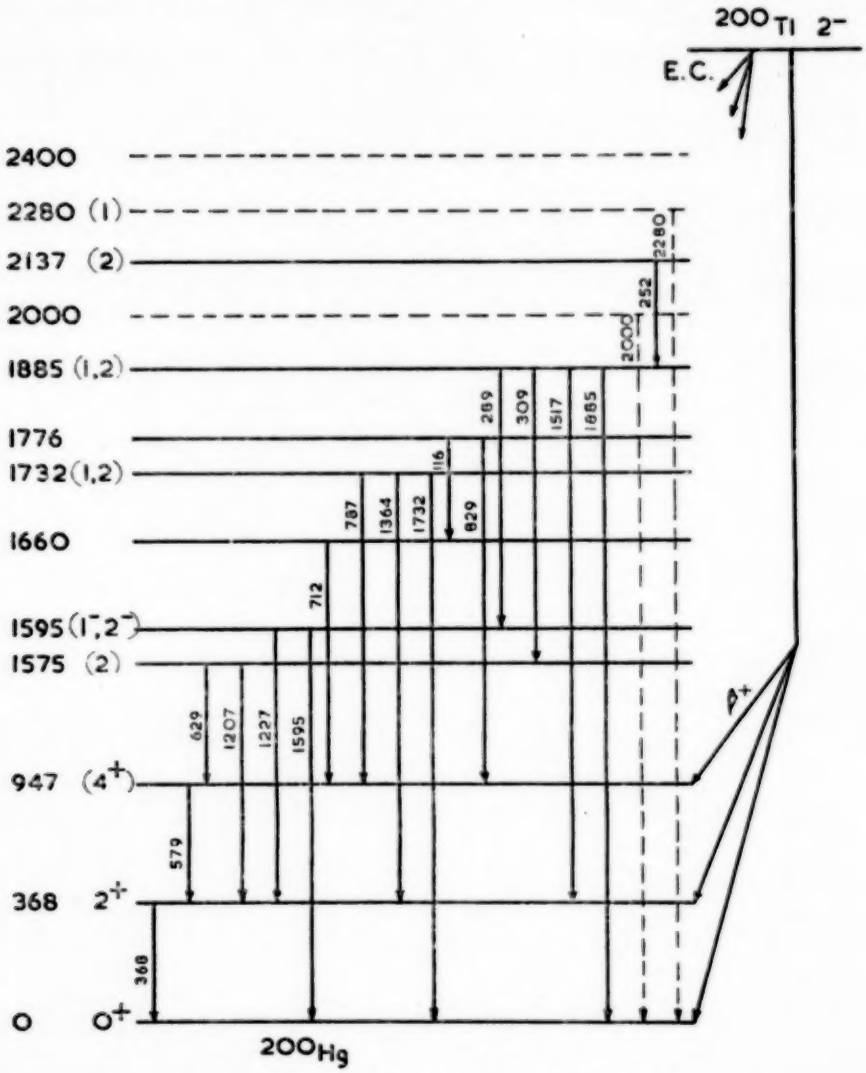

Fig. 3. The decay scheme of ^{200}Tl.

A 680 keV gamma ray is listed in table 1 and a gamma ray of this energy has been observed in the neutron capture gamma ray spectrum. Only an upper limit can be given for the intensity of this gamma ray in the decay of ^{200}Tl, since its existence is not completely certain. It occurs in a part of the complex gamma ray spectrum that is difficult to resolve in an unambiguous manner.

Therefore no conclusions can be drawn with respect to the origin of this doubtful gamma ray in the decay of ^{200}Tl.

The 1575 keV level, fed by electron capture from ^{200}Tl, mainly decays by the 1207 keV transition. Since no cross-over from this level is seen in the present work, a spin of 2 or 3 is indicated.

The 1595 keV level in ^{200}Hg is populated by negaton decay from ^{200}Au, by electron capture from ^{200}Tl and it is also observed in the capture gamma ray work. The ground state transition from this level presents a difficulty. It has been reported by Segel [6]) to be between 1.5 and 2 times as strong as the stop-over transition to the 0.37 MeV level. Herrlander *et al.* [1]) did not observe the conversion electrons corresponding to the 1595 keV transition. This led Segel [6]) to assign E1 character to the ground state transition. However, it is already evident from the gamma ray spectrum of ^{200}Tl, published in fig. 4 of ref. [1]) and in fig. 14 of ref. [7]), that there is no 1595 keV gamma ray with the required intensity in the decay of ^{200}Tl. In fact, from the decay of ^{200}Au, an intensity ratio of only 0.02 ± 0.01 has been found for the cross-over to stop-over transition [5]). Since it has been established [4, 5]) that it is the 1595 keV and not the 1575 keV level that is mainly, or even exclusively, populated from ^{200}Au, it might be deduced that it is the 1575 keV and not the 1595 keV level that is involved in the neutron capture gamma ray work. However, this solution is again unacceptable, firstly because no gamma ray of such an intensity has been observed earlier [1, 7]), secondly because the conversion electrons corresponding to the ground state transition could not have been missed by Herrlander *et al.* [1]). It must then be assumed that yet a third level in ^{200}Hg at approximately the same energy is populated by the strong 6.44 MeV capture gamma ray; its spin would then be 1. The spin of the level at 1575 keV has been determined as 2 from angular correlation work [7]), that of the 1595 keV level is probably 1^- or 2^- as shown by its population from ^{200}Au [4, 5]). Although E2 character has been assigned to the 1227 keV gamma ray by Herrlander *et al.* [1]) on the basis of the K-conversion coefficient $(4 \pm 1) \times 10^{-3}$, it should be remarked that this value of α_K is equally well explained by assuming a mixture of $\approx 85\,\%$ E1 and $\approx 15\,\%$ M2. The relatively high log *ft*-value for the allowed capture branch from ^{200}Tl to the 1595 keV level remains unexplained.

In the neutron capture gamma ray work the 1660 keV level is found to decay by a cascade of 1290 and 370 keV gamma rays, while in the decay of ^{200}Tl it is reported to decay by a 712 keV gamma ray to the 947 keV level [1]) (not resolved in the present work).

The 1732 keV level is excited both in the electron capture decay of ^{200}Tl and by neutron capture in ^{199}Hg and a cross-over gamma ray is observed in the present work as well as in the neutron capture gamma ray work. However, in the electron capture decay of ^{200}Tl, this level mainly decays by 1364 and 787 keV transitions to the first and second excited states, while in the neutron

capture gamma ray work it is reported to decay mainly by a cross-over transition to the ground state. Thus a spin of 1 or 2 is indicated.

The 1776 keV level is only excited from the electron capture decay of ^{200}Tl and no cross-over is observed. A spin of 2 or 3 is suggested. An indication of spin 4 for this level from the angular correlation measurements [7]) is not quite consistent with the log *ft*-value of 7.2 calculated for the capture branch from the gamma ray intensity [2]).

The 1885 keV level is again only excited in the electron capture decay of ^{200}Tl and a weak cross-over is also seen. Hence a spin of 1 or 2 is suggested. The conversion data given in table 1 suggest an E3 or a mixed dipole+quadrupole assignment for the 1517 keV transition of which E3 is inconsistent with the spin assignment of 1 or 2. The existence of a 2000 keV level has been derived from the direct and the summing spectrum; it can be identical with the 2020 keV level reported by Adyasevich *et al.* [10]) from neutron capture gamma ray work. The 2137 keV level is excited by both modes of excitation and no cross-over transition has been observed. This suggests a spin assignment of 2. The new level at 2280 keV seems to decay mainly by a cross-over transition. A spin of 1 is suggested. The 2400 keV level interpreted from the highest energy sum peak may be identical with the 2390 keV level reported in the neutron capture gamma ray work.

6. Conclusion

A significant amount of information about the level structure of ^{200}Hg has been derived from the study of the electron capture and positon decay of ^{200}Tl, as performed especially by Herrlander and Gerholm [1]) and by Lindqvist [7]). The results of the present work on the analysis of the complex gamma ray spectrum are in almost complete agreement with the two earlier studies.

Many of the levels seem to be populated also in the thermal neutron capture decay of ^{200}Hg, as judged from the agreement between the energies of various capture gamma rays and those from the decay of ^{200}Tl. In several cases, however, the decay properties of the level involved in the neutron capture decay are completely different from those excited from the ^{200}Tl side. Therefore it must be concluded that the similarity between the two decay modes is only very superficial and that it warrants further study.

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ON THE DECAY OF ^{123}I

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Abstract: The gamma rays from the decay of ^{123}I, produced by alpha bombardment of enriched ^{121}Sb, have been investigated with a scintillation spectrometer. New gamma rays of 275 ± 10, 320 ± 10, 340 ± 10, 380 ± 10, 435 ± 10, 500 ± 10 and 530 ± 10 keV emitted in the decay of ^{123}I have been detected. From summing studies and from the direct gamma ray spectrum the existence of levels at 435 and 502 keV in ^{123}Te, known from Coulomb excitation, has been confirmed. In addition, the existence of new levels at 690 ± 10 and 320 ± 10 keV in ^{123}Te is inferred. New gamma rays of 275 ± 10 and 320 ± 10 keV belonging to the decay of ^{121}I have also been detected.

1. Introduction

The 13 hours ^{123}I, first identified by Mitchell, Mei, Maienschein and Peacock [1]) and subsequently by Marquez and Perlman [2]) among the iodine isotopes produced by alpha bombardment of antimony, is known [3]) to decay by electron capture to the first excited state (at 159 keV) of ^{123}Te. Mathur and Hyde [4]) identified ^{123}I as a daughter of 1.8 hour ^{123}Xe, produced by bombarding iodine with protons. Recently Mitchell, Juliano, Creager and Kocher [5]) investigated the decay of ^{123}I produced by bombarding enriched ^{121}Sb (99.4 %) with 23 MeV alpha particles. These authors report the wellknown [3]) gamma ray of 159 keV and the absence of positons.

From the Coulomb excitation studies [6]) of mass separated tellurium isotopes, levels at 502 and 434 keV have been identified in ^{123}Te. The level at 502 keV decays by a stop-over transition of 342 keV to the 159 keV state and by a cross-over transition of 502 keV to the ground state, while the 434 keV level decays by stop-over (274 keV) and cross-over (434 keV) transitions to these same states. The 104 day metastable state of ^{123}Te has been well investigated [3]); it decays by a 89 keV transition to the 159 keV state and is neither populated in the decay of ^{123}I, nor is it populated in the decay of the Coulomb excited states of ^{123}Te.

The total decay energy involved in the decay of ^{123}I to ^{123}Te, estimated according to the method of Way and Wood [7]), is ≈ 1 MeV. Since the decay energy is large enough and the measured spin [8]) of ^{123}I is $\frac{5}{2}$, one might expect the excitation of the 502 and 434 keV levels, known from the Coulomb excita-

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tion studies, in the electron capture decay of ^{123}I. From a comparison with the known levels [3]) of ^{125}Te one might even expect the excitation of levels other than those known from Coulomb excitation. The study of ^{123}I was undertaken with the aim of checking these expectations.

2. Source Preparation

The preliminary studies of ^{123}I were made on the active iodine samples produced by bombarding antimony sulphide (Sb_2S_3) with 45 to 50 MeV alpha particles. The target was dissolved in 6*N* NaOH, 20 mg of iodine was added as carrier. The solution was made weakly acidic with nitric acid, oxydized with sodium nitrite and the iodine was extracted by carbon tetrachloride. In subsequent experiments enriched antimony sulphide (95.55 % ^{121}Sb †) was used as target. The active iodine, separated chemically, was investigated with a scintillation spectrometer.

3. Experimental Procedure and Results

3.1. THE DIRECT GAMMA RAY SPECTRUM

The direct gamma ray spectrum from the active iodine sources, chemically separated from the bombarded natural antimony targets, was examined immediately after chemical separation ($\approx$ 2 hours after the end of the bombardment) with a cylindrical well type NaI(Tl) crystal (diameter 62 mm, height 62 mm) optically coupled to a DuMont 6363 photomultiplier. The spectra were displayed on a 100 channel RIDL analyser. The source was placed at a distance of $\approx$ 20 cm. No important summing is expected in this geometry. The gamma ray spectrum showed the presence of ^{121}I, ^{123}I and ^{124}I (inferred from the presence of 213, 159 and 605 keV gamma rays). The decay of the gamma ray spectrum was studied at regular intervals over a period of three days; an analysis of the various spectra gave a faint indication of gamma rays of about 300, 440 and 500 keV, belonging to ^{123}I. Since their intensity was only very low ($\approx$ 5 %) compared to the contribution of the gamma rays from ^{124}I in this energy region, the gamma ray spectra from active iodine sources produced by bombarding enriched ^{121}Sb were examined in the identical geometry with the same scintillation spectrometer.

The first spectrum was taken immediately after chemical separation ($\approx$ 2 hours after the end of the bombardment) and subsequent spectra were taken at intervals of $\approx$ 2 hours in the beginning and at larger intervals later over a period of three days. The first gamma ray spectrum in the energy region from

† The enriched antimony sulphide (95.55 % ^{121}Sb, 4.45 % ^{123}Sb; other impurities are less than 1 %) was supplied from the Atomic Energy Research Establishment, Harwell, England.

100 to 700 keV is shown in fig. 1. The contribution due to ^{124}I was small at the start of this series of measurements; this contribution could easily be taken into account owing to its long half-life (4.1 days).

Each gamma ray spectrum was analyzed by successively peeling off the pulse height distribution due to the various component gamma rays starting at the high energy end. Photopeaks at 159, 213, 275, 320, 340, 380, 435, 510, 530 and

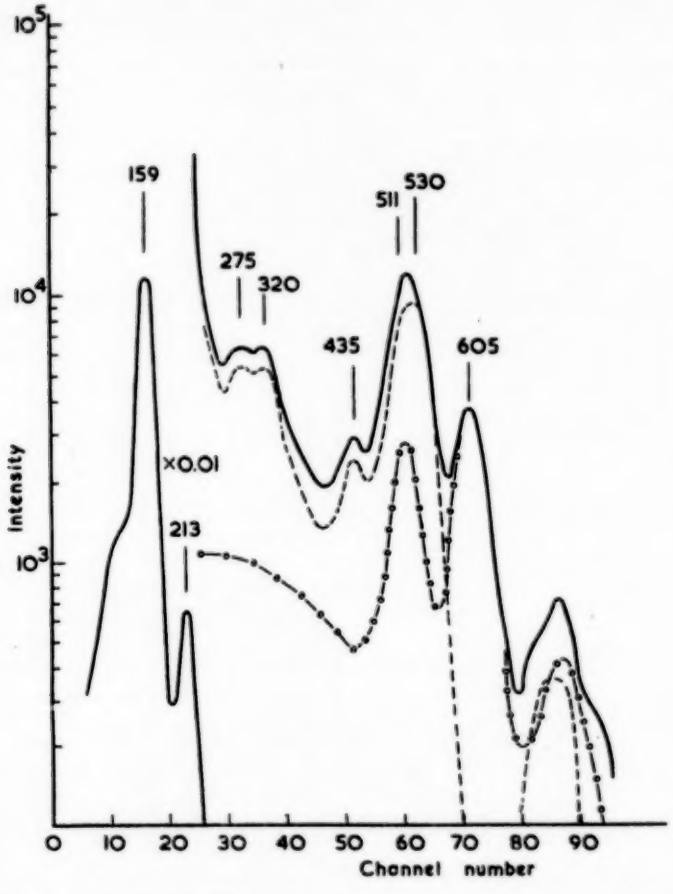

Fig. 1. Direct gamma ray spectrum of iodine sample produced from enriched ^{121}Sb in an energy region from ≈ 100 to ≈ 700 keV; the line with dots represents the contribution due to ^{124}I, the dashed line that due to ^{121}I and ^{123}I.

605 keV were resolved. The spectra from sources of ^{139}Ce, ^{203}Hg, ^{51}Cr, ^{198}Au, ^{85}Sr and ^{137}Cs were used for energy calibration and also to construct the line shapes of the various peaks resolved in the gamma ray spectrum by interpolation. The photopeaks of 159, 340, 380, 435 and 530 keV were mostly due to ^{123}I and the photopeak at 213 keV was due to ^{121}I, while photopeaks at 275, 320 and 510 keV were due to both ^{121}I and ^{123}I. The analysis of the first direct gamma ray spectrum (measured ≈ 2 hours after the end of bombardment) is summarized in table 1. No other high energy peak (up to ≈ 1400 keV) of appreciable intensity and belonging to ^{123}I was found. The intensities and

energies of the gamma rays from ^{123}I are given in table 2; the information regarding the gamma rays of ^{121}I is summarized in table 3.

TABLE 1

The analysis of the gamma ray spectrum of an iodine sample produced from enriched antimony

Gamma ray energy (keV)	Mass assignment on the basis of half-life
159	123
213	121
275±10	123 (40 %), 121 (60 %)
320±10	123 (25 %), 121 (75 %)
340±10	weak, mostly due to 123
380±10	weak, mostly due to 123
435±10	123
510±10	123 (25 %), 121 (75 %)
530±10	123
605	124

TABLE 2

Energies and relative intensities of the gamma rays following the decay of ^{123}I

Gamma ray energy (keV)	Relative intensity
159	100
275±10	0.14±0.03
320±10	0.12±0.03
340±10	0.16±0.03
380±10	0.07±0.03
435±10	0.44±0.09
500±10	0.28±0.06
530±10	2.0±0.3
(690)	< 0.08

TABLE 3

Energies and relative intensities of the gamma rays following the decay of ^{121}I

Gamma ray energy (keV)	Relative intensity
213	100
275±10	≈3
320±10	≈7
$\gamma^{\pm}$	≈20

3.2. THE SUMMING SPECTRUM

In order to obtain information regarding the possible cascade nature of the gamma rays involved in the decay of ^{123}I, the iodine source produced by bombarding enriched antimony was also examined inside the well of the well-type crystal. The first spectrum was measured immediately after chemical separation ($\approx$ 2 hours after the end of the bombardment) and the subsequent spectra were taken at intervals of $\approx$ 2 hours in the beginning and at intervals

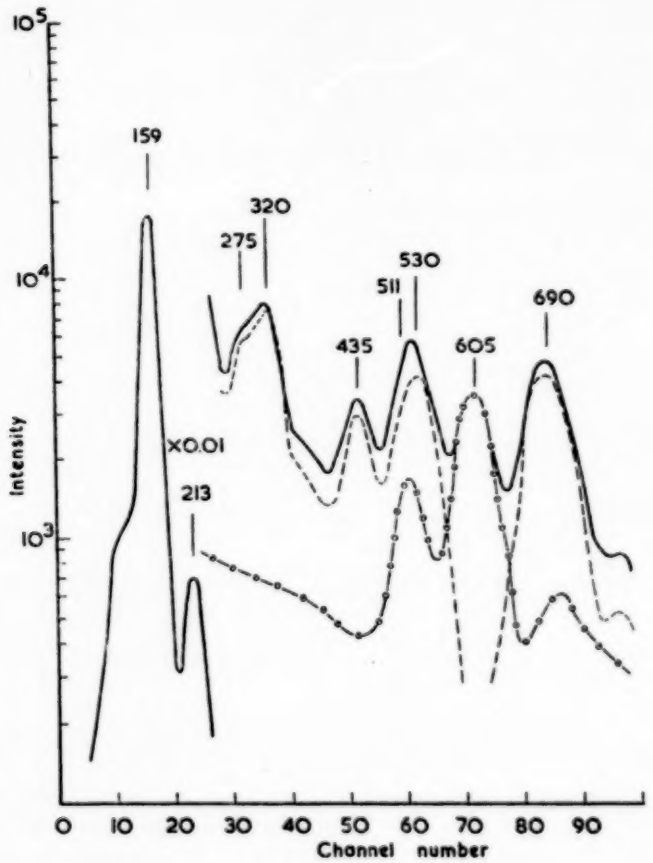

Fig. 2. Summing spectrum of iodine sample in an energy region from $\approx$100 to $\approx$ 700 keV; the line with the dots represents the contribution due to ^{124}I, the dashed line that due to ^{121}I and ^{123}I.

of 6 to 12 hours later on over a period of three days. The first spectrum (taken $\approx$ 2 hours after the end of the bombardment) is shown in fig. 2. A comparison of the summing spectrum with the direct gamma ray spectrum shows well defined peaks at 435$\pm$10 and 690$\pm$10 keV, while the intensity of the 530 keV peak goes down in the summing spectrum. A close comparison of the two spectra shows that 530 keV peak in the summing spectrum is broader than in the direct gamma ray spectrum, indicating the presence of a 502 keV sum peak; the 435 keV peak is only a little stronger than in the direct gamma ray spectrum.

It has been possible to identify the various peaks belonging to ^{121}I or ^{123}I from the decay of these peaks in the different spectra. There seems to be a sum peak at 320±10 keV due to ^{121}I, while the sum peaks at 435±10, 500±10 and 690±10 keV are due to ^{123}I. The result of the analysis of the summing spectra is summarized in table 4.

TABLE 4

Interpretation of the various peaks observed in the summing spectrum

Energy of the peak (keV)	Mass assignment	Interpretation
280±10	121 and 123	mostly due to single gamma rays
320±10	121 and 123	sum peak due to ^{121}I, sum peak due to ^{123}I uncertain
435±10	123	single gamma ray and sum of 274 and 159 keV gamma rays
500±10	123	sum peak of 342 and 159 keV gamma rays
530±10	123	single gamma ray
690±10	123	sum peak of 530 and 159 keV gamma rays

No annihilation radiation belonging to the decay of ^{123}I could be detected in the summing spectrum. Especially the energy region around 1.02 MeV was scanned and from these measurements an upper limit of 0.2 % can be placed on the intensity of annihilation radiation relative to the 159 keV gamma ray.

4. Construction of a Decay Scheme

The decay scheme given in fig. 3 is mainly based on the energies and the intensities determined from the direct gamma ray spectrum and from the summing studies.

The first excited state (159 keV) and the 104 day metastable state (248 keV) of ^{123}Te are well known [3]). The levels at 434 and 502 keV are known from Coulomb excitation [6]). The identification of these levels in the electron capture decay of ^{123}I confirms their existence. The evidence for the 690 keV level comes from the strong sum peak at this energy. The reduction in intensity of the 530 keV peak in the summing spectrum shows that the 690 keV level decays by a 530 keV transition to the 159 keV state. The presence of 320±10 and 380±10 keV gamma rays, together with the fact that the intensity of the 320 keV

gamma ray is larger than that of the 380 keV gamma ray, indicates the existence of a 320 keV level.

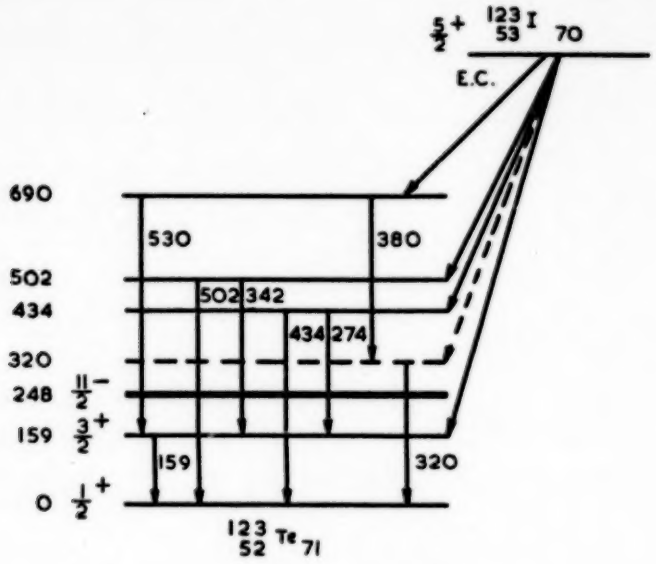

Fig. 3. Proposed decay scheme for ^{123}I.

5. Discussion

The ground state of ^{123}Te can be assigned $\frac{1}{2}^+$ from the measured spin value [9]) and from shell-model considerations. The 159 keV and the 248 keV metastable state have been the subject of many studies [3]), they have been assigned $\frac{3}{2}^+$ and $\frac{11}{2}^-$ respectively. The 502 and 434 keV levels, observed in Coulomb excitation experiments [6]), have been assigned $\frac{5}{2}^+$ or $\frac{3}{2}^+$. The spin of ^{123}I has been measured [8]) to be $\frac{5}{2}$. The log *ft*-values, calculated on the basis of the gamma ray intensities determined in the present work and assuming the value of the total decay energy of ^{123}I to be 1000 keV, are given in table 5 for the capture branches to the various levels.

TABLE 5

Log *ft*-values for the capture branches to the various levels in ^{123}Te

Level energy in keV	log *ft*
159	5.0
434	6.8
502	6.8
690	5.7

The log *ft*-value for the capture branches to both the 502 and 434 keV levels is 6.8, a rather high value for allowed capture decay. Spins of $\frac{3}{2}^+$ or $\frac{5}{2}^+$, as

suggested from Coulomb excitation experiments [6]), are also indicated by the branching ratio from these levels. The log *ft*-value of 5.7 for the capture branch to the 690 keV level indicates an allowed transition. The fact that the intensity of the cross-over transition from this level is negligible compared to that of the stop-over transition of 530 keV to the first excited state may indicate an assignment of $\frac{7}{2}^+$ or may be $\frac{5}{2}^+$ for the 690 keV level. The log *ft*-value for the capture branch to the 320 keV level has not been calculated, since the intensity of the capture branch to this level is rather uncertain, because of large uncertainties in the intensities of 320 and 380 keV gamma rays and of the possible existence of a 160 keV stop-over gamma ray from this level to the 159 keV level.

It has been suggested [6]) that the 159 keV level is the $\frac{3}{2}^+$ rotational level of the anomalous band based upon the $\frac{1}{2}^+$ ground state and that either the 434 or the 502 keV level is the $\frac{5}{2}^+$ member of this band. It then follows that the 690 keV level does not lie at the predicted position ($\approx$ 850 keV) to be the $\frac{7}{2}^+$ member of this band. The value of the parameter $\hbar^2/2I$, calculated on the above assumption is between 55 and 60 keV, i.e. approximately a factor two smaller than that for the adjacent even-even nucleides (93 keV for ^{122}Te, 101 keV for ^{124}Te). The same situation was found to exist [10]) in the cases of the nucleides ^{103}Rh, ^{107}Ag and ^{109}Ag, where the value of $\hbar^2/2I$ was also found to be between 55 and 62 keV.

However, the relative log *ft*-values for the electron capture branches do not favour this interpretation.

The last difficulty also prevents interpreting the 690 keV level as the $\frac{5}{2}$ member of the ground state rotational band and either the 434 or 502 keV state as the $\frac{3}{2}$ member, though this would yield the more reasonable values: $\hbar^2/2I = 98$ or 103 keV. The assignment of the 159 keV state as the $\frac{3}{2}$ member, instead of the 434 or 502 keV state, could agree with the log *ft*-values and would lead to a not unreasonable value: $\hbar^2/2I = 80$ keV; the fourth, $\frac{7}{2}^+$ member of this band would then lie at 1060 keV and thus could not be excited in the decay of ^{123}I.

Finally, the log *ft*-values would suggest that the 434 and the 502 keV states are the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ levels connected with the ground state rotational band. This would yield a situation which is closely analogous to that [10]) in ^{103}Rh, ^{105}Ag and ^{107}Ag and the values derived for the decoupling and moment of inertia parameters are quite reasonable. The relatively strong branching from these two levels to the 159 keV level, which is not a member of the rotational band according to this interpretation, sheds doubt upon the correctness of this assignment.

It would seem, therefore, that the possible occurrence of a rotational band in the ^{123}Te has not yet been established with reasonable certainty.

The relative intensities of the 213 keV gamma ray and the annihilation radiation from ^{121}I are in agreement with the interpretation of the decay of this nucleide by the Nuclear Data Group [3]).

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A SEARCH FOR PARITY MIXTURES IN NUCLEAR STATES

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Abstract: The gamma radiation emitted from nuclear states which are polarized by beta decay can show a 90 degree asymmetry if parity conservation in nuclear states does not hold strictly. A beta-gamma angular-correlation experiment yields linearly the quantity $\mathscr{R}\mathscr{F}$, where $\mathscr{F}$ is the relative amplitude of the irregular wave function with respect to the regular one and $\mathscr{R}$ is the matrix-element ratio of the gamma transitions. The measurement are performed with the beta-gamma decays of $A^{41} \rightarrow K^{41}$, $Xe^{133} \rightarrow Cs^{133}$, and $Lu^{177} \rightarrow Hf^{177}$ and result in the following upper limits for $\mathscr{F}$: 6×10^{-5} for A^{41}, 2×10^{-4} to 8×10^{-6} for Xe^{133}, and 3×10^{-3} for Lu^{177}.

1. Introduction

Initiated by the discovery that parity is not conserved in weak interactions the question has been considered [1]) whether in strong interactions parity conservation holds strictly. Several experiments to study this question have been carried out by Tanner [2]), Garwin *et al.* [3]), Wilkinson [4]), Heer *et al.*[5]), Segel *et al.* [6]), Jones *et al.* [7]), and Blumenfeld *et al.* [8]). Nuclear reaction experiments [2,4,6]) in particular gave the result that parity is conserved in nuclear states to a degree of 10^{-2} to 10^{-4} in the relative amplitude $\mathscr{F}$ of the irregular part of the wave function. From studies of elementary processes [3,5,7]) it follows that $\mathscr{F} \leqq 4.5\times10^{-2}$ to 2×10^{-3}, except, however, in ref. [8]).

Experiments on a greater number of nuclei are desirable to make it certain that the parity states are pure to a high degree. This is because in a single experiment accidental cancellations may occur which suppress the irregular part. The purpose of this communication is to extend the studies on parity mixture of nuclear states.

The failure of parity conservation in nuclear reactions or nuclear levels, if it exists at all, may be observed by looking for an interference effect of the parity conserving and parity-non-conserving part of the wave function. Such an interference gives rise to an odd Legendre polynomial in an experiment involving angular distributions. A parity-non-conserving coupling, therefore, generates a 90° asymmetry in the intensity distribution of a transition, as well as a longitudinal polarization of a reaction particle or a circular polarization of a photon.

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If one studies the angular distribution of a reaction particle, one measures, not the amplitude, but rather the square of the amplitude of the parity-non-conserving part. This is because in an experiment of this sort the parity violation occurs twice successively, first producing a polarization of the intermediate nucleus, secondly producing a 90° asymmetry in the angular distribution of the secondary particle from the partly polarized nucleus. The upper limit for the irregular amplitude $\mathscr{F}$, obtained by Wilkinson [4]) in an experiment of this sort, is 10^{-2}.

The answer from an angular correlation experiment can be improved greatly by producing a high degree of nuclear polarization by other means. With the help of the parity-non-conserving beta decay one can obtain a large polarization of the excited state after beta decay. A subsequently emitted gamma ray then will exhibit an angular distribution with odd powers of $\cos\theta$ with respect to the beta particle if the parities of the nuclear states involved are not pure. This experiment yields *linearly* the *amplitude* of the irregular parity part. Recently Krüger [9]) has calculated this effect.

We shall describe here several experiments in which we have studied the beta-gamma angular-distribution and obtained new estimates on the upper limit of the irregular parity admixture of nuclear states.

In a beta-gamma correlation experiment the $P_1(\cos\theta)$-term is, in general, due to the interference of parity-changing and parity-non-changing gamma radiations. The size of the coefficient of the P_1-term is given by the product of two factors $\mathscr{R}\mathscr{F}$, where $\mathscr{F}$ is the unknown relative strength of the parity-non-conserving coupling in a nuclear state in question and $\mathscr{R}$ is the matrix-element ratio of the multipole transition of order L' from the irregular part of the wave function with respect to the multipole transition of order L from the regular part of the wave function. The quantity $\mathscr{R}$ can be estimated in each case on the basis of a nuclear model.

Krüger [9]) has derived the following expression of the anisotropy A for a gamma transition $I_f \to I_{ff}$ following an allowed beta decay $I_i \to I_f$:

$$A = \frac{v}{c}\,\frac{1-2\lambda\sqrt{I_f(I_f+1)}}{1+\lambda^2}\,\frac{I_f(I_f+1)-I_i(I_i+1)+2}{\sqrt{3I_f(I_f+1)}}\,F_1(L, L', I_{ff}, I_f)\mathscr{R}\mathscr{F}, \quad (1)$$

where v is the electron velocity, $\lambda = c_V\int 1/C_A\int\sigma$ is the ratio of Fermi to Gamow-Teller amplitudes, and F_1 is a geometrical coefficient tabulated in ref. [10]). The anisotropy A is as ususal defined by

$$W(\theta) = 1+A\cos\theta, \quad (2)$$

where θ denotes the angle between beta particle and gamma ray.

In section 2 we describe the experimental method and in section 3 we proceed to discuss the experimental results.

2. Experimental Method

The apparatus for the study of the $\beta-\gamma$ angular correlation is shown schematically in fig. 1. A lucite vacuum chamber of 20 cm diameter, 50 cm height, and 0.4 cm wall thickness is used. The beta detector is an anthracene crystal (3.8 cm diameter, 0.4 cm thick) mounted onto the face of a RCA 6810A photo-

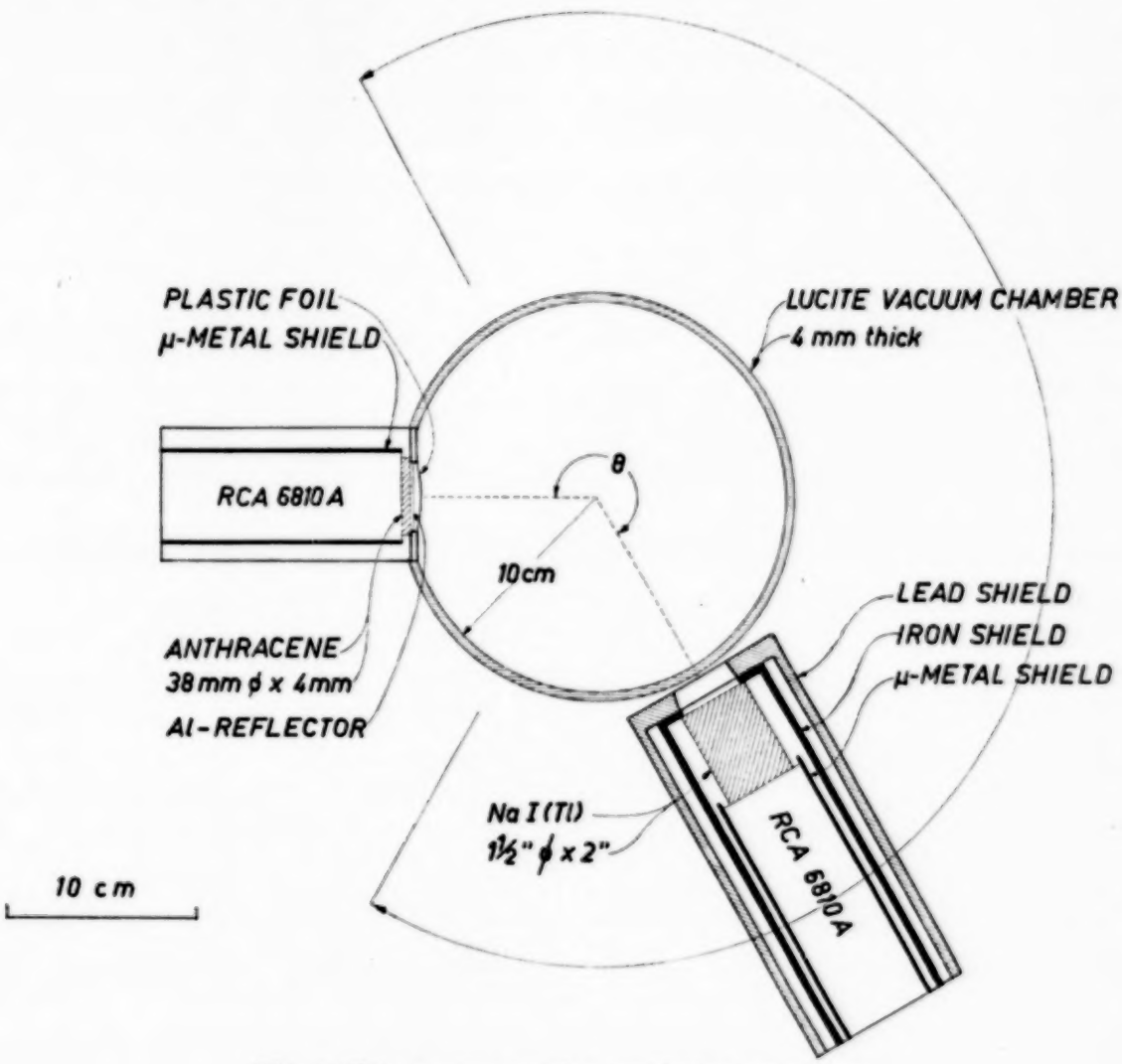

Fig. 1. Beta-gamma angular correlation apparatus.

multiplier tube. A 2.2 mg/cm² aluminium foil serves as a light reflector. The beta particles leave the vacuum chamber and enter the beta detector through a 5 mg/cm² plastic foil (Hostaphan) of 3.5 cm diameter. The gamma detector is a NaI(Tl) crystal (3.8 cm diameter, 5 cm high) mounted onto the face of a RCA 6810A photomultiplier tube, which is shielded magnetically by μ-metal and 0.3 cm iron. A 1 cm lead shield is provided to reduce scattering between the detectors. A further discrimination against scattered quanta is obtained by narrow photopeak discrimination as discussed below. The gamma detector is

rotated automatically between $\theta = 60°$ and $\theta = 300°$ in steps of 30° or 60° at predetermined time intervals of 4 to 20 minutes. The timing clock also stops the scaling circuits and activates the printing registers during the time in which the detector moves to the next position.

The electronic circuit is a Bell-type fast-slow coincidence apparatus. Its optimum resolving time is about 1 ns for gamma-ray energies above 200 keV. This is performed by cutting off the limiter pentodes (E180F), (which require a cut off voltage of 1.25 V) with photomultiplier anode pulses of about 150 V so that the effective cut off time will be small when compared with the phosphor decay time. Because of the finite lifetime of the nuclear states in question the resolving time actually used in the experiments reported in section 3 is $2\tau = 7$ ns. The number of accidental fast-slow triple coincidences is always less than 1 % of the coincidence counting rate.

The slow pulses are picked up from dynode No. 10 of the 14 stage photomultipliers, amplified by non-overload amplifiers Hamner N-302, analyzed with Fairstein single channel analyzers, and counted with Hewlett-Packard scalers 520A which are followed by slow scalers and printing registers.

The channel width of the gamma-ray single-channel analyzer is about one half-width of the scintillation photo peak, thus suppressing Compton quantum contribution. From time to time the coinciding beta and gamma spectrum is measured by a 100 channel pulse height analyzer whose fast gate is triggered by the fast-slow coincidence signal. This gives a good check on the presence of disturbing gammas and betas or conversion electrons of the isotope under investigation or of contaminations. In every run the beta singles, gamma singles, fast coincidences, and the slow coincidences are checked for following the right half-life. The room is temperature stabilized to 0.5° C. Small local temperature variations of a few tenths of a degree between different angular positions of the γ-counter are found to be present. Their influence on the single and coincidence counting rates is studied carefully and appropriate small corrections are applied.

For the analysis of the results the slow triple coincidences are divided by the single gammas and single betas for every measuring interval. Decay corrections are applied.

The performance of the apparatus is checked with Tm^{170} and Co^{60}. Fig. 2 shows the well known velocity dependent $P_2(\cos\theta)$ angular distribution of the first-forbidden beta-decay of Tm^{170} and the isotropic distribution obtained with Co^{60}.

The search for a 90° asymmetry is carried out with A^{41}, Xe^{133} and Lu^{177}. The source strengths are of the order of 0.1 mc. The A^{41} is produced either in the cyclotron of the Max Planck Institut Heidelberg or in the reactor of the University of Frankfurt (Main) †. The gas with an initial activity of about

† We thank Dr. Schmidt-Rohr, Max Planck Institut Heidelberg, and Dr. Fiebiger, Physikalisches Institut der Universität Frankfurt (Main), for the preparation of the sources.

0.3 mc/cm³ is transferred into a glass sphere of 0.7 cm radius and 7 mg/cm² wall thickness. The cyclotron produced activity contains a small amount of gaseous contamination with short half-life, which is negligible in the beta counting rate after the first hour. The accepted energy range of the single

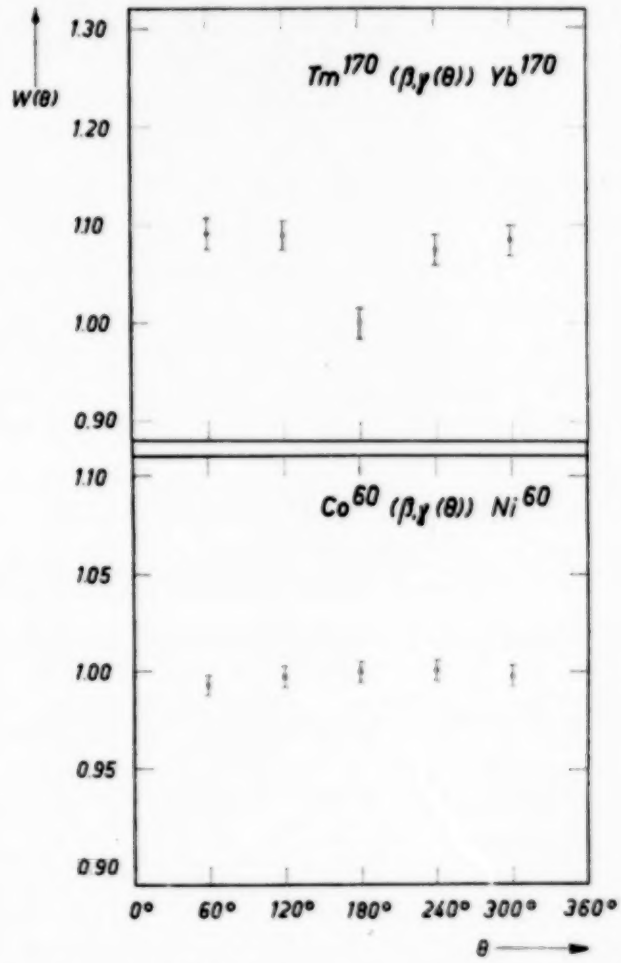

Fig. 2. Check of the performance of the beta-gamma angular correlation apparatus with Tm^{170} and Co^{60}. Owing to the first forbidden charakter of the Tm^{170} beta decay a $P_2(\cos\theta)$-term is present. For Co^{60} the photopeaks of both gamma rays are accepted; corrections for gamma-gamma coincidences are made.

channel analyzer in the beta channel extends from 0.4 to 1.2 MeV. 12 sources are investigated with a total measuring time of 44 hours, whereby the gamma detector changes its position every 4 minutes.

The Xe^{133} is produced in the reactor of Harwell, England, with a specific activity better than 1 c/cm³. The fresh Xe^{133} source contains approximately 2 % Xe^{133m} and 0.5 % Xe^{131m}. A few mm Hg of the gas are transferred into a glass sphere of 0.35 cm radius and 7 mg/cm² wall thickness. The total measuring time is 190 hours, during which the gamma detector changes its position every 10 minutes. In a 50 hours run beta particles are accepted with energies ranging

from 255 keV to 340 keV. In the remaining time the energy interval lies between 180 keV and 340 keV. In the latter case a contribution of internal or external bremsstrahlung to the 81 keV photo peak is smaller than 5×10^{-4}, which was found by measuring the angular correlation after moving the gamma single-channel-analyzer from 81 keV to 120 keV. This is in accordance with a theoretical estimate.

The Lu^{177} is produced in the Harwell reactor with a specific activity of about 10 mc/mg. Because of the elapsed time between production of the source and begin of the measurements a contamination of 3.7 h Lu^{176m} does not influence the measurements. The Lu_2O_3 sources are spread over an area of about 0.2 cm^2 on a collodium film and covered with a second one. The films are supported on an aluminium ring of 3.9 cm outer diameter, 3.8 cm inner diameter, and 0.07 cm thickness. The single channel analyzer accepts beta particles between approximately 90 keV and 180 keV. The total measuring time is 180 hours, in which the gamma detector changes its position every 20 minutes.

3. Results and Discussions

3.1. $A^{41}(\beta,\gamma(\theta))K^{41}$, M2—E2 INTERFERENCE

The decay of A^{41} is summarized in fig. 3a [11]). The allowed beta decay excites a state at 1.29 MeV in K^{41} which has a measured half-life of 6.7×10^{-9} sec [12]). From this half-life the M2 character of the radiation was deduced [12]), inferring a spin $\frac{7}{2}-$ for the excited state. This spin assignment as well as the measured

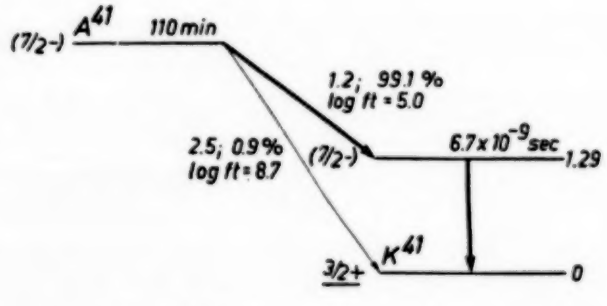

Fig. 3a. Decay scheme $A^{41} \rightarrow K^{41}$.

ground state spin of $\frac{3}{2}+$ are in accord with the shell model predictions. The A^{41} spin $\frac{7}{2}-$ is derived from the beta decay *ft*-values [13]) to the ground state and to the excited state respectively and is also in agreement with the shell model.

We now estimate the value of $\mathscr{R}$ for the gamma transition. Here we are dealing with a M2—E2 interference. According to the single particle estimate [14]) one finds a matrix element ratio for E2 and M2 transitions for the mass 41

$$\langle E2\rangle/\langle M2\rangle = 4.$$

The M2 gamma transition with $\tau_{\frac{1}{2}} = 6.7 \times 10^{-9}$ sec is slower than the single-particle estimate by a factor of 63. A survey [9]) of the E2 transition probabilities in the region of mass-number 40 indicates that the E2 transition is expected to be enhanced by a factor of at least 5 compared to the single-particle estimate. Summarizing these arguments and taking into account that parity mixture can occur in both excited and ground state, we get

$$\mathscr{R} = 4 \times \sqrt{63} \times \sqrt{5} \times 2 = 142.$$

From this, eq. (1) furnishes, with $v/c = 1$,

$$A = 0.2 \mathscr{R} \mathscr{F} = 28 \mathscr{F}.$$

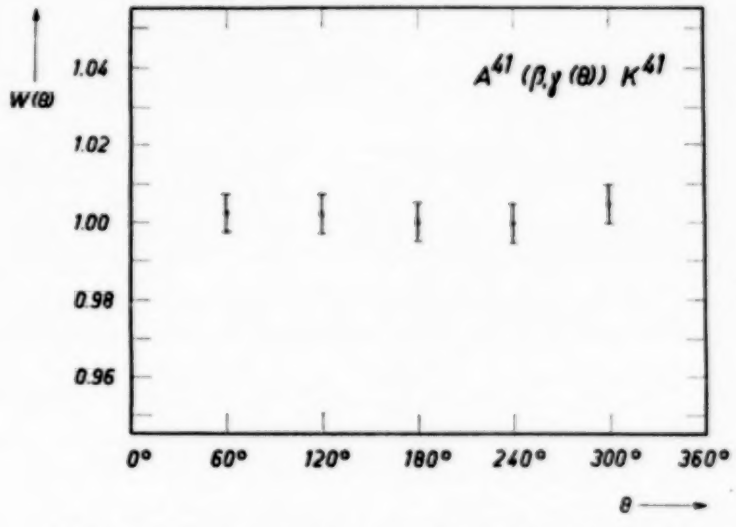

Fig. 3b. Beta-gamma angular distribution $A^{41} \rightarrow K^{41}$.

The value of λ was assumed to be 0, i.e. pure G.T. interaction. Recently Mayer-Kuckuck and Nierhaus, Max Plank Institut für Kernphysik, Heidelberg, determined $\lambda = -0.36$ from beta-gamma circular polarization measurements. Therefore the anisotropy A is given by $A = 95 \mathscr{F}$.

A least square analysis for $P_1(\cos\theta)$ of the experimental data (corrected for the finite angular resolution of the counters and the finite source extension) contained in fig. 3b, neglecting $P_2(\cos\theta)$ due to contributions from second forbidden beta transitions, gives the result

$$A = 0.0024 \pm 0.0036.$$

From this result one concludes that

$$\mathscr{F} \leqq 6.2 \times 10^{-5},$$

the $\leqq$ sign meaning that the quoted value corresponds to the upper error limit.

3.2. $Xe^{133}(\beta, \gamma(\theta))\, Cs^{133}$, M1—E1 INTERFERENCE

The decay of Xe^{133} is illustrated in fig. 4a [11, 15]). The main branch of the allowed beta transition leads to a state at 0.081 MeV which has a measured half-life of 6.3×10^{-9} sec [16]). A spin of $\frac{5}{2}+$ has been assigned to the 0.081 MeV state. All spin assignments of fig. 4a in Xe^{133} and Cs^{133} are in agreement with shell model predictions. From the conversion coefficient and gamma-gamma

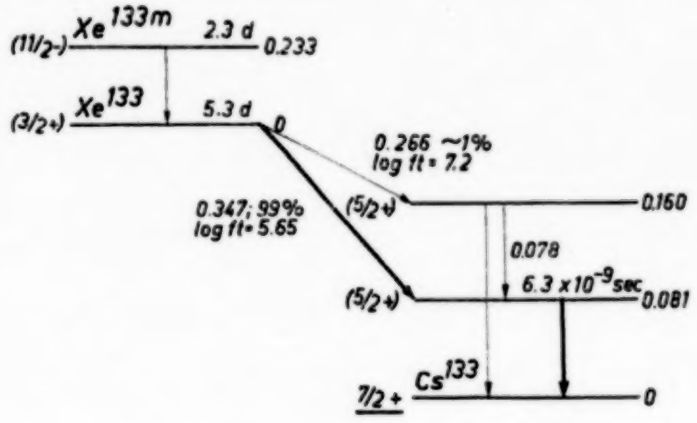

Fig. 4a. Decay scheme $Xe^{133} \rightarrow Cs^{133}$.

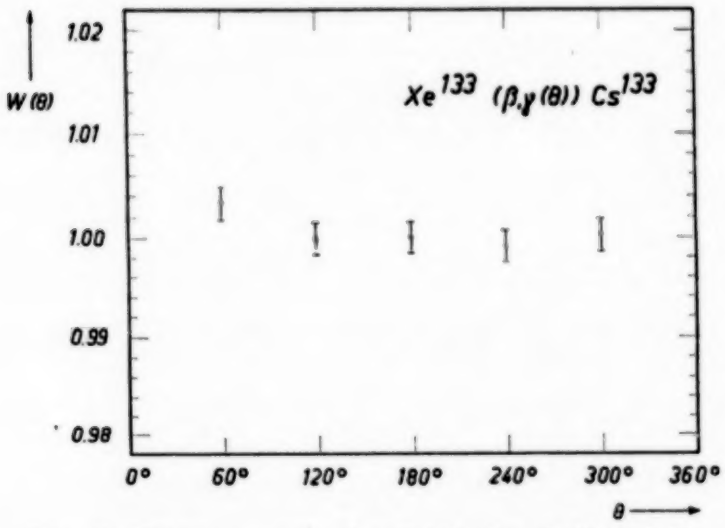

Fig. 4b. Beta-gamma angular distribution $Xe^{133} \rightarrow Cs^{133}$.

angular correlation [16]) it follows that the 81 keV level of Cs^{133} decays by nearly pure M1 radiation (E2 $\leqq$ 2.5 %) †. The experimental M1 transition probability corrected for internal conversion is lower than the single particle value by a factor of 650. A conservative estimate of $\mathscr{R}$ can be made assuming that the

† About 1 % of the beta transitions reach a level at 160 keV, having probably spin $\frac{1}{2}+$ [15, 16]). The 78 keV line depopulating this level with an intensity of about 0.8 % is also detected in this experiment.

competing E1 is slowed down by the same factor as the M1 transition†; then it follows that

$$\mathscr{R} = 2\langle \mathrm{E1}\rangle/\langle \mathrm{M1}\rangle = 23.$$

This yields the following result from eq. (1), with $v/c = 0.7$:

$$A = 0.7\mathscr{R}\mathscr{F} = 16\mathscr{F}.$$

The experimental value obtained from the data contained in fig. 4b is††

$$A = 0.0015 \pm 0.0015,$$

which results in

$$\mathscr{F} \leqq 1.9 \times 10^{-4}.$$

The E1 transition probability might conceivably be larger since the Cs nucleus is close to a single particle nucleus. In the earlier experiments mentioned in section 1 the estimated upper limits of $\mathscr{F}$ have been obtained throughout by using for the nuclei in question the single particle value for the calculation of $\mathscr{R}$. If we also use the single particle estimate, $\mathscr{R}$ assumes the value 585, which yields $A = 405\mathscr{F}$ and

$$\mathscr{F} \leqq 7.5 \times 10^{-6}.$$

3.3. Lu^{177} $(\beta, \gamma(\theta))$ Hf^{177}, E1—M1 INTERFERENCE

Fig. 5a contains the decay scheme of Lu^{177} [15]). Besides the intensive first forbidden beta decay to the ground state of Hf^{177} and a weak first forbidden transition of 3 % to the first excited state which belongs to the ground state

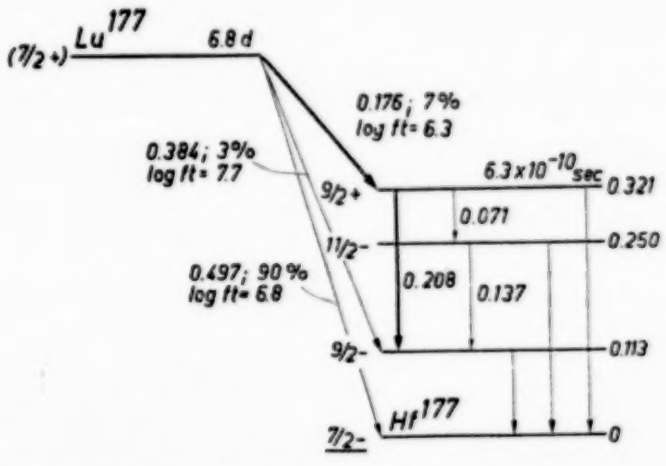

Fig. 5a. Decay scheme $Lu^{177} \rightarrow Hf^{177}$.

† No E1 lifetime is known experimentally in the region of mass-number 130. In the region of strongly deformed nuclei above mass 150 the tendency for stronger E1 retardation is towards heavier mass-numbers. It should be pointed out that our assumed retardation factor for the parity-violating E1 transition is even nearly one order of magnitude greater than the measured value for e.g. Sm^{152}.

†† Account has been taken of the plural scattering of electrons in the wall of the glass bulb.

rotational band, there is an allowed transition of 7 % to the 321 keV $\frac{9}{2}+$ level of Hf^{177}. This level decays by the E1 radiations of 71 keV, 208 keV, and 321 keV with intensity ratios 2 : 220 : 3. From the measured half-life of 6.3×10^{-10} sec [17]) it follows that the prominent E1 radiation of 208 keV is retarded by a factor of 10^5 when compared with the single particle value. This is due to the intrinsic structure of the strongly deformed nucleus. Therefore an E1—M1 interference can be expected if there is parity non-conservation.

A rough estimate for $\mathscr{R}$ is given by Krüger [9]) for the decays $Yb^{175}(\beta, \gamma(\theta))Lu^{175}$ and $Lu^{177}(\beta, \gamma(\theta))Hf^{177}$ by estimating the possible admixture of the lower collective state to the upper state of same spin but different parity. For Hf^{177} the estimate results in a rather unfavourable matrix element ratio of $\mathscr{R} = 4.1$, so that

$$A = 0.6\mathscr{F}.$$

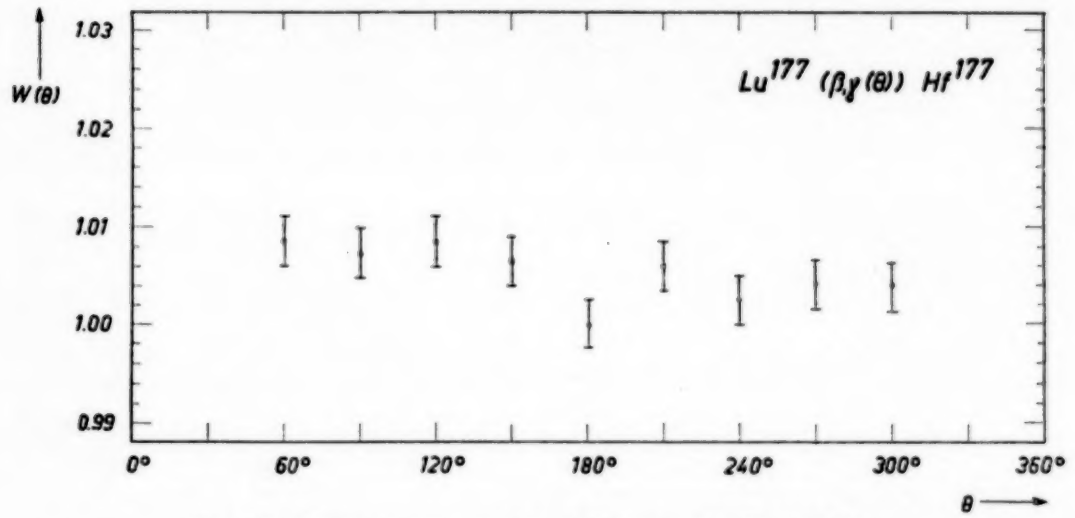

Fig. 5b. Beta-gamma angular distribution $Lu^{177} \rightarrow Hf^{177}$.

The experimental data are contained in fig. 5b. A weak $P_2(\cos\theta)$ contribution is due to coincidences between the 208 keV gamma ray and the conversion electrons of the 113 keV transition ($\alpha_K = 0.75$; K/L/M/ = 2/4/1 [15]) †. A least square analysis †† for the Legendre polynomials $P_1(\cos\theta)$ and $P_2(\cos\theta)$ yields ††† for A

$$A = 0.00035 \pm 0.00130.$$

† The contribution of L and M conversion electrons, which cannot be well separated energetically from the beta particles because of the poor energy resolution of the anthracene crystal, is verified by measuring the coincidence spectrum of the beta detector with the gated 100 channel pulse height analyzer as described in section 2.

†† According to Klema [19]), the attenuation of the angular correlation involving the 113 keV state is small. Since the 113 keV state has about the same lifetime as the 321 keV state we have neglected an attenuation effect in the analysis.

††† A 90° asymmetry produced by 208 keV internal bremsstrahlung of the ground-to-ground-state beta transition is approximately one order of magnitude smaller than the experimental limit of error.

From this value one concludes

$$\mathcal{F} \leqq 2.8 \times 10^{-3}.$$

In conclusion we may state that the experimental upper limits found for the amplitude $\mathcal{F}$ of the parity violating wave function, $\mathcal{F} \leqq 2.8 \times 10^{-3} \ldots 7.5 \times 10^{-6}$, are at least two orders of magnitude greater than the value to be expected according to Gell-Mann [18]) for the existence of a parity non-conserving nuclear force from the (np)(np) term of the weak coupling.

It is a pleasure to thank Professor B. Stech and Mr. L. Krüger for many valuable discussions. In particular we thank Mr. L. Krüger for making available to us his calculations prior to publication. We are indebted to Mr. G. Knissel and Mr. R. Neubert for assistance during the experiments. Financial support of this work has been granted by the Bundesministerium für Atomkernenergie and the Badisch-Württembergisches Wirtschaftsministerium and is gratefully acknowledged. One of us (F. B.) wishes to thank Professor O. Haxel and Professor J. H. D. Jensen for their kind hospitality.

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K-MESON PRODUCTION IN π—p COLLISIONS AND K-MESON PARITY

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Abstract: The reactions $\pi^+ + p \to p + K^+ + \overline{K}^0$ and $\pi^- + p \to p + K^- + K^0$ are considered and numerical values of the differential cross-sections given. The calculations have been done with a two-fold purpose: (i) on the Gell-Mann coupling scheme, with the hope of deciding between the two cases of even and odd relative K—N parity and (ii) on the recent suggestion of a KKπ-interaction by Pais (requiring odd $K^+ - K^0$ relative parity) with a view to test its existence.

1. Introduction

The theoretical attempts to understand the baryon and K-meson interactions have, so far, been on the Gell-Mann [1]) interaction scheme. In this scheme Σ, Λ, N have the same (assumed) parity and the $\Sigma - \Lambda$ mass difference is neglected. Further, K^+ and K^0 are assumed to form an isobaric doublet and consequently their relative parity $p(K^+, K^0)$ is even. The question of the relative K-meson-hyperon parity $p(K, Y)$ is left open.

Recently Pais [2]), in an attempt to relax the strong symmetries of the Gell-Mann model, has suggested a scheme of strong interactions. In the Pais scheme all the baryons appear as isobaric doublets (as before) but the K^+ and K^0 as isobaric singlets. Again, Σ, Λ, N have the same parity. Pais [2]) introduces two new quantum numbers S_1 and S_2 such that $S_1 + S_2 = S$, the usual strangeness number. On requiring separate conservation of S_1 and S_2 he finds that certain observed reactions (for example, $\pi^+ + p \to \Sigma^+ + K^+$) appear to be forbidden by S_1 (or S_2) conservation, S being conserved. To circumvent this difficulty Pais suggests, since K^+ and K^0 are isobaric singlets, that $p(K^+, K^0)$ be odd and thus introduces a direct parity conserving, but S_1 (or T_3) violating, KKπ-interaction of the form $f(2M_K)$ $(K^- K^0 \pi^+ + K^0 K^+ \pi^-)$, where f is the dimensionless coupling constant and M_K the K-meson mass.

In this paper the reactions

$$\pi^- + p \to p + K^- + K^0, \qquad \text{(a)}$$

$$\pi^+ + p \to p + K^+ + K^0, \qquad \text{(b)}$$

have been considered. Both the reactions conserve strangeness but are S_1-violating on the Pais scheme. The cross-section calculations have been carried

out with a two-fold purpose: (i), on the Gell-Mann scheme for both p(K, Y) even and odd, with the hope of distinguishing between them; (ii) using the KKπ-interaction, with a view to test its existence. Similar calculations have, recently, been carried out for the production of K-mesons from nucleon-antinucleon annihilation [3]).

2. Calculation

The reactions given above have been calculated to the lowest order using covariant perturbation theory. Since the effective coupling constants are smaller than in the pion-nucleon case the perturbation calculation may not be unreliable.

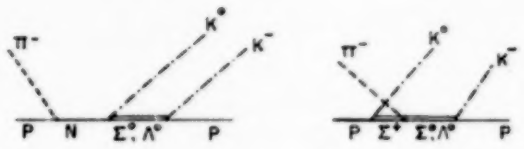

Fig. 1.

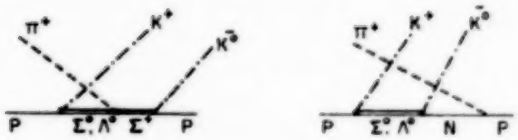

Fig. 2.

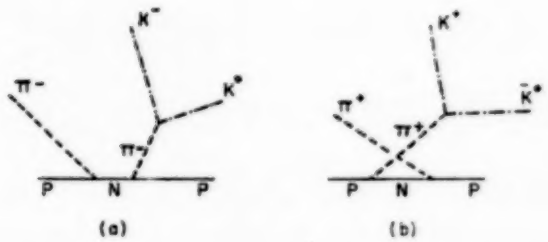

Fig. 3.

Figs. 1 and 2 show the lowest order Feynman graphs for the reactions (a) and (b) respectively, while fig. 3 shows the graphs when they occur through the KKπ-interaction.

A point to be noticed is that if the $\Sigma-\Lambda$ mass difference is neglected, and the coupling constants G_Σ and G_Λ for [NKΣ] and [NKΛ] interactions are taken to be equal, all the graphs in figs. 1 and 2 will give zero result on inclusion of both Λ^0 and Σ^0 in the intermediate state. To get a finite result we take $G_\Sigma \neq G_\Lambda$ but neglect the $\Sigma-\Lambda$ mass difference. We assume, of course, that Σ, Λ and N have the same parity and are fermions.

The calculation is straightforward except for the complication owing to three outgoing particles. It was not found possible to obtain an exact analytic expression for the angular distribution of the charged K-meson. Numerical integration was considered undesirable and consequently the exact differential cross-sections have been given for fixed values of certain variables, chosen from the viewpoint of the experimentalist.

The calculation has been made in the laboratory frame. In this frame the threshold [4]) is 1.36 GeV. All energies have been expressed in terms of M_K, the K-meson rest mass. An incident pion beam of energy 10 M_K (4.96 GeV) is considered as some experiments [5]) have already been done in this energy region. The pion rest mass has been neglected and the nucleon rest mass taken to be $2M_K$. The direction of the neutral K-meson has been fixed along the incident direction (also the z-axis). This choice has the advantage that the cross-sections become independent of the azimuthal angles. Further, to facilitate experimental observation and identification of the reaction the results for only low energy outgoing proton and charged K-meson are given. This is necessary, as a high energy K^+ may be confused with a low energy proton track. Also in the $\pi^+ + p$ reaction the identification of the K^+ is not enough, as it may come from $\pi^+ + p \to \Sigma^+ + K^+$. Since, the charged K-meson and the proton have low energy, the neutral K-meson will have large kinetic energy and one may neglect its rest mass with little error. This enables an easy solution of the energy-momentum conservation equations.

3. Results

On fixing the direction of the neutral K-meson the differential cross-section is obtained as a function of θ_1, the angle with the z-axis and $|\mathbf{k}_1|$ the momentum of the charged K-meson. In table 1 we give the values of θ_1, $|\mathbf{k}_1|$ and θ_p and $|\mathbf{p}|$, the corresponding variables for the outgoing proton, to help in identifying the reaction. The last six columns, give in mb the value of $d\sigma/d|\mathbf{k}_1|d(\cos\theta_1)d(\cos\theta_2)$ for the fixed value of $\theta_2 = 0$, the angle of the neutral K-meson with the z-axis. Estimates show that the cross-sections will not appreciably alter for θ_2 lying in a cone of one degree about the forward direction. The numbers for the differential cross-section have to be multiplied by $c_0 = (G^2/4\pi)^2 f^2/4\pi$ in the $KK\pi$-case ($p(K^+, K^0)$ odd) and by $c'_0 = G^2/4\pi(G_\Sigma^2/4\pi)^2(1-G_\Lambda^2/G_\Sigma^2)^2$ in the other two cases, where G and f are the pion-nucleon and the $KK\pi$-coupling constants.

In the table 1 only low values of $|\mathbf{k}_1|$ have been chosen as for higher values the neglect of the neutral K-meson mass is not a good approximation. One would nevertheless expect similar results for other values of $|\mathbf{k}_1|$ in the region $0 \leqq |\mathbf{k}_1| \leqq M_K$.

TABLE 1

Charged K-meson		Outgoing proton		$\pi^- + p \to p + K^- + K^0$			$\pi^+ + p \to p + K^+ + \overline{K}^0$		
Momentum	Angle with z-axis	Momentum	Angle with z-axis	$p(K^+, K^0)$ even		$p(K^+, K^0)$ odd	$p(K^+, K^0)$ even		$p(K^+, \overline{K}^0)$ odd
$\lvert\mathbf{k}_1\rvert$	θ_1	$\lvert\mathbf{p}\rvert$	θ_p	$p(K, N)$ $-$ $(\times 10^{-3})$	$p(K, N)$ $-$ $(\times 10^{-2})$	$(\times 10^{-3})$	$p(K, N)$ $+$	$p(K, N)$ $-$ $(\times 10^{-1})$	$(\times 10^{-2})$
0.25	180°	2.423	0	0.076	0.348	0.379	0.752×10^{-2}	0.187	0.592
	120°	1.988	6° − 18′	0.097	0.411	0.596	0.467×10^{-2}	0.137	0.622
	90°	1.631	8° − 50′	0.117	0.435	0.903	0.312×10^{-2}	0.116	0.672
	60°	1.321	9° − 36′	0.140	0.405	1.373	0.221×10^{-2}	0.096	0.747
	0	1.033	0	0.132	0.297	2.122	0.164×10^{-2}	0.075	0.876
0.5	180°	5.044	0	0.118	0.318	0.391	6.19×10^{-1}	1.63	1.936
	120°	3.028	8° − 16′	0.228	0.885	1.028	0.785×10^{-1}	1.24	2.008
	90°	2.032	14° − 18′	0.370	1.34	2.446	0.283×10^{-1}	0.926	2.236
	60°	1.355	18° − 41′	0.536	1.33	5.695	0.139×10^{-1}	0.592	2.680
	0	0.756	0	0.718	0.426	14.374	0.081×10^{-1}	0.345	4.284
0.75	90°	2.771	15° − 47′	0.574	1.77	3.071	0.851×10^{-1}	1.68	3.876
1	90°	4.098	14° − 10′	0.624	1.31	1.785	0.298×10^{-1}	1.52	7.52

The values of the variables are given in the laboratory system for a pion beam of momentum $10 M_K$ (4.96 GeV) incident in the z-direction. (M_K is the K-meson rest mass.) The last six columns give the differential cross-section $d\sigma/d|\mathbf{k}_1|d(\cos\theta_1)d(\cos\theta_2)$ in mb, for the fixed value $\theta_2 = 0$ of the angle of the neutral K-meson with the z-axis. The symbol $p(\ ,\ )$ refers to relative parity. The cross-sections must be multiplied by the relevant coupling constants in each case.

4. Discussion

To simplify the discussion, the results have been plotted in figs. 4, 5 and 6.

Fig. 4 shows the angular distribution of K^- for the π^-+p reaction. For $|\mathbf{k}_1| = 0.25$ and $p(K, N)$ even the curve is not shown as the numbers are too small. For $p(K, N)$ odd $|\mathbf{k}_1| = 0.25$, the distribution is practically isotropic but shows some indication of being symmetric about 90°. This feature is clearly apparent for $|\mathbf{k}_1| = 0.5$. For $p(K, N)$ even and $|\mathbf{k}_1| = 0.5$ there is slight increase in the forward direction. The curves for the KKπ-case, $p(K^+, K^0)$ odd, show an increase in the forward direction for both values of $|\mathbf{k}_1|$. The cross-sections for the three cases, when multiplied by the coupling constants c_0, c'_0 (see section 3), would still be distinguishable in the neighbourhood of $|\mathbf{k}_1| = 0.5$. Taking $G^2/4\pi \approx 15$, $f^2/4\pi \approx 10^{-2}$ (see ref. [2])) we have, $c_0 \approx 2.3$ and $c'_0 \approx 15$, if we assume $G_\Sigma{}^2/4\pi \approx 0.5$ and [3,6]) $G_A = \sqrt{3}G_\Sigma$.

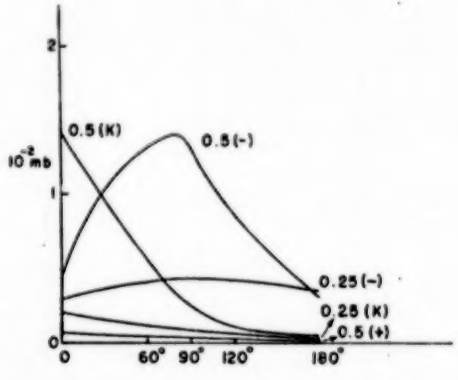

Fig. 4. The differential cross section $d\sigma/d|\mathbf{k}_1|d(\cos\theta_1)d(\cos\theta_2)$ (in 10^{-2} mb) for the π^-+p reaction, is plotted against θ_1 (the abscissa). The signs (+), (−) and (K) refer to the columns 4, 5 and 6 of table 1 and the numbers indicate the value of $|\mathbf{k}_1|$. The cross-sections plotted must be multiplied by the relevant coupling constants in each case.

The numbers for the π^++p reaction are generally larger than for the π^-+p reaction. This may possibly be due to the destructive interference between the two amplitudes for $T = \frac{3}{2}$ and $T = \frac{1}{2}$, in the π^-+p reaction, giving a smaller result than for the π^++p reaction, for which only the $T = \frac{3}{2}$ amplitude is available.

The angular distribution of K^+, for the π^++P reaction, is shown in figs. 5 and 6 for the values $|\mathbf{k}_1| = 0.25$ and 0.5, respectively. In fig. 5 both cases, $p(K, N)$ odd or even, show backward preponderance, while the KKπ-case is nearly isotropic with a slight forward increase. Fig. 6 shows similar but more pronounced characteristics. For example, the even $p(K, N)$ case shows a definite backward peaking. Although the two cases $p(K, N)$ even or odd show similar angular distributions, the magnitudes of the cross-sections are different enough to permit distinction between the two cases. Further, it would be possible to distinguish either of these from the KKπ-case.

As remarked earlier, one would expect similar angular distributions in the neighbourhood of $|\mathbf{k}_1| = 0.5$. Thus an experimental study of the two reactions would be of great help in deciding the question of K-meson parity. It should be kept in mind, however, that our results are true only for the lowest order.

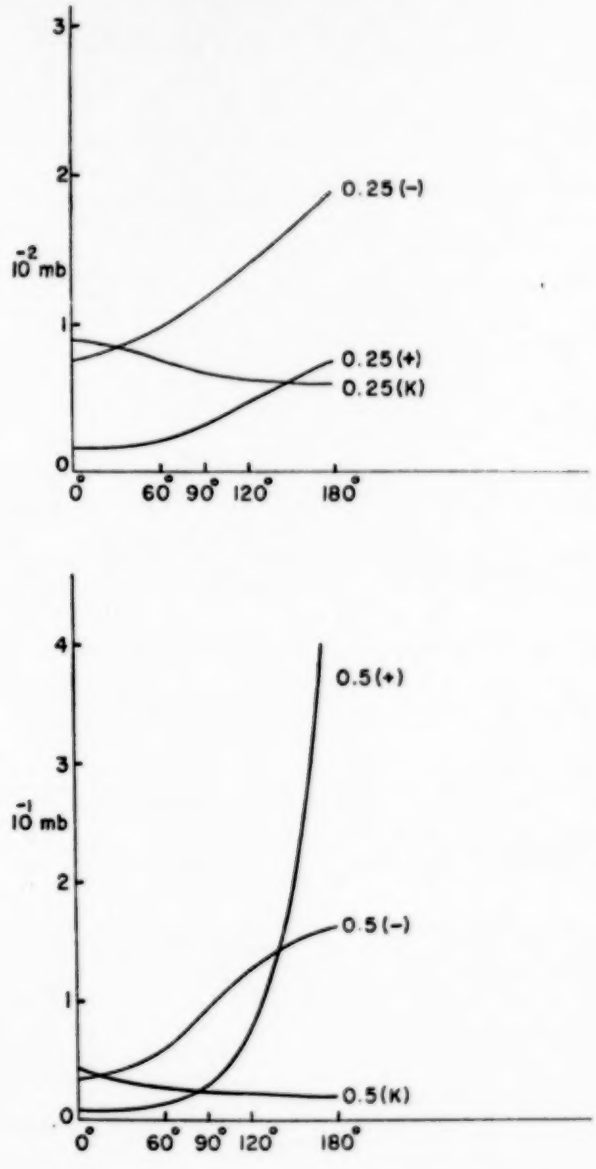

Fig. 5 and 6. The differential cross-section $d\sigma/d|\mathbf{k}_1|d(\cos\theta_1)d(\cos\theta_2)$ for the $\pi^+ + p$ reaction is plotted against θ_1 (the abscissa). The signs (+), (−) and (K) refer to the columns 7, 8 and 9 of table 1 and the numbers indicate the value of $|\mathbf{k}_1|$. The cross-sections plotted must be multiplied by the relevant coupling constant in each case.

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COMPLEX PHASE SHIFTS IN PROTON-PROTON SCATTERING

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Abstract: The imaginary parts of the p—p phase shifts at 380 MeV are deduced from pion production data; their effect on elastic scattering appears to be negligible. Anomalies at the π^++d threshold are also very small.

1. Introduction

Scattering phase shifts become complex when inelastic processes are possible, making analyses more difficult in general. However, the imaginary parts of the p—p phase shifts can already be inferred from pion production experiments, for energies not far above threshold. We calculate these imaginary phases at 380 MeV, and estimate their importance. They appear to be negligible in the description of elastic scattering.

The "Wigner cusps" at the p+p → π^++d threshold ($\approx$ 290 MeV) are also too small to detect.

2. General Remarks

The p—p scattering amplitude is a matrix in spin space. Each element can be decomposed into partial wave amplitudes of the form

$$R_j = (\exp 2i\Delta_j - 1)/2i \tag{1}$$

where j is a label; Δ_j is the phase shift, which we write in terms of its real and imaginary parts:

$$\Delta_j = \delta_j + i\gamma_j. \tag{2}$$

The properties of scattering involve the phase shifts only through expressions like $\mathrm{Re}(R_j R_k^*)$ and $\mathrm{Im}(R_j R_k^*)$. To separate the contributions of real and imaginary parts, it is convenient to introduce the quantities $\beta_j = \frac{1}{2}\{1-\exp(-2\gamma_j)\} \approx \gamma_j$(for small γ_j). We now find

$$\mathrm{Re}(R_j R_k^*) = \sin\delta_j \sin\delta_k \cos(\delta_j-\delta_k) + \beta_j \sin\delta_k \sin(\delta_k-2\delta_j) + \beta_k \sin\delta_j \sin(\delta_j-2\delta_k) + \beta_j\beta_k \cos 2(\delta_j-\delta_k), \tag{3}$$

$$\mathrm{Im}(R_j R_k^*) = \sin\delta_j \sin\delta_k \sin(\delta_j-\delta_k) + \beta_j \sin\delta_k \cos(\delta_k-2\delta_j) - \beta_k \sin\delta_j \cos(\delta_j-2\delta_k) + \beta_j\beta_k \sin 2(\delta_j-\delta_k). \tag{4}$$

The inelastic p—p cross-section is well known to be

$$\sigma_1 = (\pi/2k^2) \sum_{J,\alpha} (2J+1)(1-|\exp 2i\Delta_{J\alpha}|^2), \tag{5}$$

where k is the relative momentum, J is the total angular momentum and α represents all other appropriate quantum numbers.

It is obvious that σ_1 is more sensitive than elastic scattering to the imaginary phases, for it depends on them alone. If the phases are small, we also notice that the elastic and inelastic cross-sections may be comparable even when the imaginary phases are much smaller than the real ones, i.e. $\gamma = O(\delta^2)$. In this situation eq. (3) is dominated by the first term, which contains real phases only. The imaginary phases are important in eq. (4), however, for the first three terms are of the same order.

In p—p scattering [1]), the elastic differential and total cross-sections, triple-scattering parameters and spin-correlation parameters (unpolarized beam) depend exclusively on the expressions (3). Terms like (4) only appear in the polarization, and in less accessible quantities which have never yet been measured. We might therefore expect the polarization to be the most sensitive to small imaginary phases.

Lastly we remark that in the Born approximation R_j is replaced by Δ_j (itself approximate). If $\Delta_j = \delta_j$ is real, this amounts to adding an imaginary phase of order δ^2, which makes little difference in eq. (3) but a big difference in eq. (4). In fact $\mathrm{Im}(R_j R_k^*)$ now vanishes, which is why the Born approximation gives no polarization for a real potential. In Moravcsik's [2]) method of phase-shift analysis, which makes limited use of the Born approximation, there is little error, however; the approximation is only used for weakly scattered waves, which contribute mainly through interference with strongly scattered waves.

3. p—p Scattering at 380 MeV †

At energies not far above threshold, where the maximum pion momentum in the c.m. system, η, is of order $\eta \leqq 1$ (in units $m_\pi c$), a phenomenological theory of pion production in two-nucleon collisions has been developed [3, 4, 5]). If we suppose the nucleons to emerge mainly in an S state and the pion to be formed in an S or P state relative to them, and consider angular momentum and parity, we can associate the inelastic p—p processes with the $^1S_0$, $^1D_2$, $^3P_0$ and $^3P_1$ incident channels only. A study of these processes reveals how much of the inelastic cross-section comes from each channel, and so gives the imaginary phases (ignoring inelastic electromagnetic processes).

† This is the energy of the extracted proton beam at Liverpool, and so has some practical interest.

At higher energies this simple picture fails, for other final states become important. We cannot yet assess their relative importance, nor say which incident channels are most affected.

At 380 MeV proton laboratory energy, $\eta = 0.8$ † and this theory is fairly reliable. We now use it to find the imaginary phases. The most important corrections to the simple theory, of which something is known experimentally, are mentioned.

We use the notation $\delta_L + i\gamma_L$ for singlet and $\delta_L{}^J + i\gamma_L{}^J$ for triplet phases; L and J are orbital and total angular momenta. The relative momentum is $k = 2.14 \times 10^{13}$ cm^{-1}.

3.1. $^1S_0$ AND $^1D_2$ SCATTERING

In the simple theory, these channels contribute only to the final states $\pi^+ + d$ and $\pi^+ + (n+p)$ (unbound $^3S_1$). Their combined cross-section for the former is $1.01 \pm 0.08\ \eta^3$ mb [6, 7]), and the 310 MeV p—p phase shift analysis [2, 9]) shows indirectly [6, 10]) that the ratio of $^1S_0$ to $^1D_2$ contributions lies approximately in the interval 0.05—0.15. The cross-section for unbound final states can be estimated from theory; Gell-Mann and Watson [4]) tabulate the percentage of transitions to these states, which agree fairly well with experiment [4, 7]). The figure for 380 MeV is 29 %. Hence we estimate the $^1S_0$ cross-section to be 0.07 mb and $\gamma_0 = 0.28°$, within about 50 %. Similarly we take the $^1D_2$ cross-section to be 0.66 mb and $\gamma_2 = 0.55°$, within about 10 %.

Going beyond the simple theory, $^1S_0$ and $^1D_2$ can give pion production with the nucleons in relative P-states. The cross-section for such processes is ill-determined, but may be as high as $0.12\ \eta^6$ mb [8, 11, 12]). If this value is taken, and the whole effect attributed to $^1S_0$, the increase in γ_0 is large but less than 50 %. The maximum correction to γ_2 is only 5 %.

3.2. $^3P_0$ SCATTERING

In the simple theory, this channel contributes equally to the $\pi^0 + 2p$ and $\pi^+ + n + p$ states. The cross-section for these appears to be $0.05 \pm 0.02\ \eta^2$ mb [8, 11]), though some argue it may be more [12]); hence $\gamma_1{}^0 = 0.13°$ within about 40 %.

Beyond the simple theory, the ^3P and ^3F incident channels can lead to pion production with the nucleons in P-states. The cross-section for these processes is uncertain [8, 11, 12]) but may be of the order of $0.1\ \eta^8$ mb. If it were as large as this and $^3P_0$ were mainly responsible, $\gamma_1{}^0$ would be increased by 50 %.

3.3. $^3P_1$ SCATTERING

This channel contributes to the final states $\pi^+ + d$ and $\pi^+ + (n+p)$ (unbound $^3S_1$), with a measured cross-section for the former $0.138 \pm 0.015\eta$ mb [6, 7]). The ratio of bound to unbound transitions should be as in subsection 3.1;

† We ignore the pion mass differences, etc., and take the same η for all processes. The error introduced is unimportant in the present work.

we estimate an overall cross-section 0.16 mb, and $\gamma_1{}^1 = 0.22°$, within about 10 %. The additional processes mentioned in subsection 3.2 cannot increase this by more than 10 %.

3.4. DISCUSSION

The imaginary phases γ_2 and $\gamma_1{}^1$ are determined quite well, but γ_0 and $\gamma_1{}^0$ with much less accuracy. We can distinguish two sources of uncertainty: experimental error and failure of the simple theory. The former can presumably be reduced by more careful measurements. The latter is more serious, for corrections to the simple theory cannot be elucidated without a much more detailed study of the production processes, requiring new experiments of greater difficulty.

The real parts of the p—p phase shifts are not yet known at 380 MeV. However, for the sake of illustration, let us take them from solution 1 of the 310 MeV analysis [2, 9]) and consider the expressions (3) and (4) both with and without the calculated imaginary phases. Using the values $\delta_2 = 13°$, $\delta_1{}^0 = -14°$, $\delta_1{}^2 = 16°$, we have

$$\mathrm{Re}(R_2 R_2{}^*) = 0.0497 \text{ or } 0.0506, \tag{6a}$$

$$\mathrm{Re}(R_1{}^0 R_1{}^{2*}) = -0.0573 \text{ or } -0.0577, \tag{6b}$$

$$\mathrm{Im}(R_1{}^0 R_1{}^{2*}) = 0.0338 \text{ or } 0.0333. \tag{6c}$$

The first figure in each case is for the complex phase shift. These terms are typical among the larger ones of physical importance; for example (6a) contributes 17 % of the total elastic "nuclear" cross-section. The imaginary phases would cause bigger relative corrections if the associated real phases were smaller, but each term would also be smaller and of less physical importance.

The relative correction in (6c) is not much larger than that in (6a) or (6b), in spite of our expectations in section 2. In the case of (6a), this is because the ratio of real to imaginary phase is much larger for $^1D_2$ than for $^3P_0$. To explain (6b) we note that the real phases concerned are not truly small, so that the sine and cosine of $(\delta_1{}^2 - 2\delta_1{}^0)$ are nearly equal instead of differing by an order of magnitude.

The corrections in (6) are only a few percent. Since the quantities of experimental interest also contain other terms which suffer no correction, we may expect them to be even less sensitive. For example, the total elastic "nuclear" cross-section is altered by only 0.7 %. With the current level of experimental uncertainty (perhaps 3 % in the differential cross-section but certainly more in other measurements), the imaginary phases would seem to be negligible in elastic p—p scattering at 380 MeV.

At higher energies the imaginary phases are more important but also less well determined. At 500 MeV ($\eta = 1.3$) they are up to five times as large in the

simple theory. However, using the estimates in subsections 3.1—3.3, γ_2 is uncertain to 30 % and the remaining three are uncertain within factors of three to ten. Other partial waves may also be significantly absorbed.

4. The π^++d Threshold

The foregoing considerations also show that the "Wigner cusps" [13]) (see ref. [14]) for recent literature) in elastic p—p scattering at this threshold ($\approx$ 290 MeV) are very small.

The anomalous behaviour derives from the $^3P_1$ channel. If δ_t is the real phase shift at threshold, the complex phase shift close above is approximately † $\delta_t+i\alpha\eta$, ($\alpha = 0.15\pm0.02$ degrees: see subsection 3.3); by appropriate analytic continuation around the branch point at threshold [15]), the real phase shift close below is $\delta_t-\alpha|\eta|$. Physical quantities now contain terms linear in η or $|\eta|$ near threshold, and their energy derivatives are singular there. The anomaly in the total elastic nuclear cross-section is a "rounded step" rather than a cusp, if δ_t lies in the fourth quadrant as suggested by the 310 MeV analysis [2, 9]). The behaviour of other quantities is less easy to predict.

Assuming a characteristic interaction radius of roughly a pion Compton wavelength, and noting that k^{-1} is also of this order, the approximation above should be good for $|\eta| \ll 1$. Taking this to mean $|\eta| < 0.1$, (an interval 3 MeV wide, in proton laboratory energy), $\alpha|\eta| < 0.02°$ in this region. It is clear that the threshold anomalies are far too small to be detected.

5. Conclusions

The imaginary phases in p—p scattering at 380 MeV can be inferred from pion production data with moderate accuracy, and therefore add no special difficulty to a phase-shift analysis. However, their effect on elastic scattering is probably negligible, with present experimental errors. It is a pity that they cannot be found accurately at the higher energies where an appreciable effect is expected. The imaginary phases cannot be found at these energies without a much more detailed study of the pion production processes.

Anomalies in elastic scattering at the p+p $\rightarrow \pi^+$+d threshold are too small to measure.

† We ignore the Coulomb forces, which is permissible in estimating whether the nuclear effects are measurable [14]).

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THE THREE-BODY K-MESON DECAY

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Abstract: A method for calculation of polarization states of relativistic Dirac fields is given and applied to the three-body K-meson decay. The matrix elements for these processes are given the most general form compatible with a V—A-interaction and relativistic invariance. As a consequence, the final expression for the transition probabilities includes two unknown functions to be determined from the experimental material. The analysis of the results available from different sources is still found insufficient for definite conclusions regarding the type of interaction. New possible tests of the V—A-theory are proposed.

1. Introduction

The V—A theory for weak interactions developed by Sudarshan and Marshak [1]) and, independently, by Feynman and Gell-Mann [2]) has been remarkably successful in clarifying the experimental material available from weak decays. In this theory the interaction responsible for all weak processes arises from the coupling of the chiral current J^λ with itself. The current J^λ is constructed from the V—A combination of bilinear covariants according to the postulate of 'minimal chiral interaction' [3]). As a consequence of the theory there is no distinction between the electron and the muon except for the mass difference, and these particles are coupled in the same way in all weak and electro-magnetic interactions.

In the search for possible tests of the theory the three-body K-meson decay has been considered by several authors [4]) as the only process in which both the electron and the muon modes have actually been observed. The measurement of the polarization state of the outgoing muon in three-body K-meson decay would be of particular interest as this is the only known three-body process involving a muon. In two-body decays angular momentum selection rules require the 'wrong' polarization for the outgoing muon, but in three-body decays this restriction is removed and we expect the negative muon to come out left-handed and the positive muon right-handed.

The difficulties in the theoretical treatment of weak decays are due to the fact that these processes take place over virtual particle-antiparticle pairs and this introduces vertices corresponding to strong interactions. So far, the only way known to handle these difficulties is to introduce unknown functions in the

matrix element for the transition. These unknown functions can later be determined from the experimental material, as will be shown.

Polarization effects in the decays of elementary particles show up as typical correlation terms in the expression for the transition amplitude. A suitable formalism for calculation of polarization effects will be discussed first and later applied to the K-meson decay.

2. Polarization Formalism for Dirac Particles

The spin of a Dirac particle is given by the expectation value of the antisymmetrical tensor $\sigma_{\mu\nu}$. In the rest system of the particle only the space components are non-vanishing and they form a three-dimensional antisymmetric tensor which can be represented as an axial vector. This three-dimensional spin vector is identical with the ordinary Pauli spin vector known from non-relativistic quantum mechanics. A detailed discussion of these matters has been given by Stapp [5]) and will therefore be omitted here.

It is of interest to investigate to what extent the spin can be defined in an arbitrary direction for a Dirac particle. This can most easily be done by introducing the projection operator λ given by

$$\lambda = \tfrac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{\xi})$$

where $\boldsymbol{\xi}$ is a fixed unit vector and $\boldsymbol{\sigma}$ the usual three-dimensional spin vector of the particle. In order that the spin in the direction given by the vector $\boldsymbol{\xi}$ shall have physical meaning, the operator λ must commute with the Hamiltonian H for the free Dirac particle. H is given by

$$H = \boldsymbol{\alpha}\cdot\mathbf{p}+\beta m$$

and direct computation gives

$$[H,\lambda] = i\boldsymbol{\alpha}\cdot(\boldsymbol{\xi}\times\mathbf{p}).$$

This commutator vanishes if either one of the following conditions are fulfilled:

1) $\mathbf{p}$ is parallel to $\boldsymbol{\xi}$,

2) $\mathbf{p} = 0$.

From the first condition it follows that the longitudinal spin always has physical meaning. From the second condition it follows that the spin can be specified in an arbitrary direction in the rest frame of the particle. The spins of elementary particles involved in a process are therefore most conveniently defined in their rest systems by means of projection operators of the same type as the λ-operator defined above.

It often happens that the calculations of transition amplitudes are referred to the centre of mass frame or some other frame where the particle or the par-

ticles are not at rest. In this case the appropriate projection operator is obtained by a Lorentz transformation of the λ-operator, namely

$$\lambda' = L(\mathbf{p})\lambda L^{-1}(\mathbf{p}),$$

where

$$L(\mathbf{p}) = \sqrt{\frac{E+m}{2m}}\left\{1+\frac{|\mathbf{p}|}{E+m}\gamma_0\gamma_1\right\},$$

$$L^{-1}(\mathbf{p}) = \sqrt{\frac{E+m}{2m}}\left\{1-\frac{|\mathbf{p}|}{E+m}\gamma_0\gamma_1\right\};$$

$$\gamma_0 = \beta, \quad \gamma_i = \beta\alpha_i, \quad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3.$$

In the expressions for $L(\mathbf{p})$ it has been assumed that the primed system is moving along the x_1-axis of the rest system of the particle and with the axis of the two systems parallel to each other; E stands for the total energy of the particle in the primed system and $\mathbf{p}$ stands for its momentum in this system.

It can easily be shown by direct computation that the commutator $[H, \lambda']$ for the free field solutions of the Dirac particle has the eigenvalue zero, i.e. the polarization states defined by the λ'-operator have physical significance, although it is only in the rest frame of the particle that this operator has a simple interpretation.

3. Decay Distributions and Correlation Functions

We shall consider the following two processes:

$$K^- \to \pi^0 + \mu^- + \bar{\nu},$$
$$K^- \to \pi^0 + e^- + \bar{\nu},$$

described by the Feynman graphs in fig. 1.

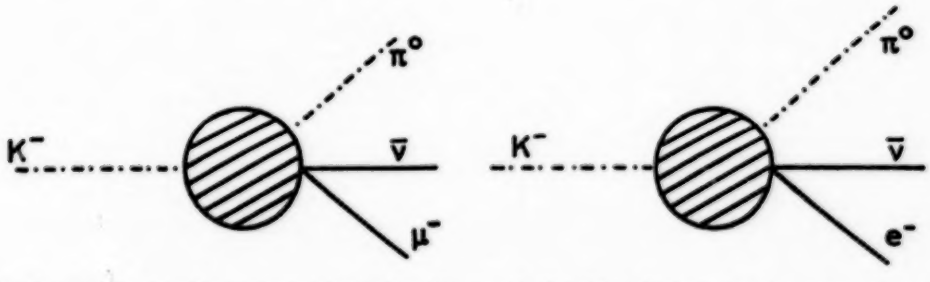

Fig. 1. Feynman graphs representing the $K_{\mu 3}$ and the K_{e3} decays in the present analysis.

Assuming a universal V—A interaction for weak processes, the interaction part of the Hamiltonian responsible for the K-meson decay can be written

$$H_w = [G_V J^\lambda + G_A J^{5\lambda}] j_\lambda + \text{h.c.}; \tag{3.1}$$

G_V and G_A denote the usual coupling constants for vector and axial vector interactions, J^λ and $J^{5\lambda}$ are vector and pseudovector currents formed by the meson field operators, and j_λ is the usual lepton current $\bar{\psi}_\mu \gamma_\lambda(1+\gamma_5)\psi_\nu$. The transition matrix element is given by

$$M = \langle \mu\bar{\nu}\pi|H_w|\mathrm{K}\rangle. \tag{3.2}$$

The pseudovector meson current does not contribute to this process, as follows from the conservation of parity at the strong interaction vertex in the graph. If further the electromagnetic corrections are neglected, the matrix element can be written

$$M = G_V\langle\pi|J^\lambda|\mathrm{K}\rangle\langle\mu|j_\lambda|\nu\rangle. \tag{3.3}$$

The meson part of the matrix element cannot be evaluated as it involves strong interaction, but its most general form follows from invariance considerations [6]). There are four energy-momentum vectors in the problem, but due to energy-momentum conservation the number of independent four-vectors is decreased by one. In the Born approximation the four-momenta of the muon and the antineutrino always appear together as the sum, and consequently the number of independent four-vectors in this approximation is two. It is convenient to keep the energy-momentum vectors of the K-meson and the π-meson independent, and the most general form of the meson part of the matrix element can be written

$$G_V\langle\pi|J^\lambda|\mathrm{K}\rangle = fk^\lambda + g(k-\pi)^\lambda \tag{3.4}$$

where k^λ and π^λ denote the four-momentum of the K-meson and the π-meson, and the two unknown functions f and g are functions fo the scalar quantity $\pi^\nu k_\nu$. From invariance under time-reversal it follows that the functions f and g can be chosen to be real functions.

The lepton part of the matrix element is given by

$$\langle\mu|j_\lambda|\nu\rangle = \sqrt{\frac{m_\mu m_\nu}{E_\mu E_\nu}}\,\bar{u}_\mu\gamma_\lambda(1+\gamma_5)u_\nu, \tag{3.6}$$

and the following form of the matrix element is obtained:

$$M = \sqrt{\frac{m_\mu m_\nu}{E_\mu E_\nu}}\,\bar{u}_\mu[fk^\lambda\gamma_\lambda + g(k-\pi)^\lambda\gamma_\lambda](1+\gamma_5)u_\nu. \tag{3.5}$$

To rewrite this expression we introduce the four-momenta of the muon and the antineutrino and denote them by p^λ and q^λ. From energy-momentum conservation it then follows that

$$(k-\pi)^\lambda = (p+q)^\lambda.$$

Using the Dirac equation for the muon and the massless antineutrino the second term in the expression for M can be rewritten

$$u_\mu(\not{p}\gamma_\lambda+\not{q}\gamma_\lambda)(1+\gamma_5)u_\nu = m_\mu u_\mu(1+\gamma_5)u_\nu \tag{3.7}$$

and (3.6) takes the form

$$M = \sqrt{\frac{m_\mu m_\nu}{E_\mu E_\nu}}\bar{u}_\mu(fk^\lambda\gamma_\lambda+gm_\mu)(1+\gamma_5)u_\nu. \tag{3.8}$$

It should be noted that the 'usual' normalization for the neutrino field has been used, but this is just a formal trick to simplify the calculations: it has no other consequences, as the neutrino mass denominators cancel out.

In the matrix element (3.8) the spinors u_μ and u_ν characterize the polarization and energy state of the outgoing leptons. To calculate correlation functions and transition probabilities we need the square of the matrix element. For these calculations it is most convenient to use the technique using λ'-operators, discussed earlier. It is also convenient to introduce the particle-antiparticle projection operators

$$\Lambda_\pm(p) = \frac{m\pm\not{p}\gamma_\lambda}{2m}.$$

If these operators are introduced simple calculations give for the square of the matrix element

$$|M|^2 = \frac{m_\mu m_\nu}{E_\mu E_\nu}\,\mathrm{tr}[O\Lambda_-(q)\gamma_0 O^+\gamma_0\Lambda_+(p)\lambda'(p)], \tag{3.9}$$

where

$$O = fk^\lambda\gamma_\lambda+gm_\mu$$

and O^+ denotes the Hermitian conjugate operator;

$$\lambda'(p) = L(\mathbf{p})\tfrac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{\xi})L^{-1}(\mathbf{p}).$$

With $m_\nu = 0$, and making use of the relation

$$\frac{m_\mu+\not{p}\gamma_\lambda}{2m_\mu} = L(\mathbf{p})\tfrac{1}{2}(1+\gamma_0)L^{-1}(\mathbf{p}),$$

(3.9) simplifies to

$$|M|^2 = \frac{1}{4E_\mu E_\nu}\,\mathrm{tr}[O\not{q}\gamma_\lambda\gamma_0 O^+\gamma_0 L(\mathbf{p})(1+\gamma_0)(1+\boldsymbol{\sigma}\cdot\boldsymbol{\xi})L^{-1}(\mathbf{p})]. \tag{3.10}$$

The factor $1/4E_\mu E_\nu$ will in the following be included in the unknown functions. The explicit calculation of the trace in (3.10) is straightforward but tedious.

One obtains

$$
\begin{aligned}
|M^2| = f^2 &\left\{(E_\mu - \mathbf{p}\cdot\boldsymbol{\xi})(2k_0 k^\lambda q_\lambda - q_0 k^\lambda k_\lambda) + \left(\frac{\mathbf{p}\cdot\boldsymbol{\xi}}{E_\mu + m_\mu} - 1\right)(2\mathbf{p}\cdot\mathbf{k}k^\lambda q_\lambda\right. \\
&\left. - \mathbf{p}\cdot\mathbf{q}k^\lambda k_\lambda) + m_\mu(2\mathbf{k}\cdot\boldsymbol{\xi}k^\lambda q_\lambda - \mathbf{q}\cdot\boldsymbol{\xi}k^\lambda k_\lambda)\right\} \\
&+ 2m_\mu fg\left\{m_\mu k^\lambda q_\lambda + E_\mu(q_0\mathbf{k} - k_0\mathbf{q})\cdot\boldsymbol{\xi} - (\mathbf{p}\cdot\mathbf{q})(\mathbf{k}\cdot\boldsymbol{\xi})\right. \\
&\left. + (\mathbf{p}\cdot\mathbf{k})(\mathbf{q}\cdot\boldsymbol{\xi}) - \frac{q_0}{E_\mu + m_\mu}(\mathbf{k}\cdot\mathbf{p})(\mathbf{p}\cdot\boldsymbol{\xi}) + \frac{k_0}{E_\mu + m_\mu}(\mathbf{q}\cdot\mathbf{p})(\mathbf{p}\cdot\boldsymbol{\xi})\right\} \\
&+ m_\mu^2 g^2\left\{E_\mu q_0 - (\mathbf{p}\cdot\mathbf{q}) - m_\mu(\mathbf{q}\cdot\boldsymbol{\xi}) + q_0(\mathbf{p}\cdot\boldsymbol{\xi}) - \frac{1}{E_\mu + m_\mu}(\mathbf{p}\cdot\mathbf{q})(\mathbf{p}\cdot\boldsymbol{\xi})\right\}.
\end{aligned}
\tag{3.11}
$$

In the case of a K-meson decaying at rest this expression simplifies considerably and one obtains

$$
\begin{aligned}
|M|^2 = m_K^2 f^2 &\left\{E_\nu E_\mu + (\mathbf{p}\cdot\mathbf{q}) - m_\mu(\mathbf{q}\cdot\boldsymbol{\xi}) - E_\nu(\mathbf{p}\cdot\boldsymbol{\xi}) - \frac{(\mathbf{p}\cdot\boldsymbol{\xi})(\mathbf{p}\cdot\mathbf{q})}{E_\mu + m_\mu}\right\} \\
&+ 2m_\mu m_K fg\left\{m_\mu E_\nu - E_\mu(\mathbf{q}\cdot\boldsymbol{\xi}) + \frac{(\mathbf{p}\cdot\mathbf{q})(\mathbf{p}\cdot\boldsymbol{\xi})}{E_\mu + m_\mu}\right\} \\
&+ m_\mu^2 g^2\left\{E_\nu E_\mu - (\mathbf{p}\cdot\mathbf{q}) - m_\mu(\mathbf{q}\cdot\boldsymbol{\xi}) + E_\nu(\mathbf{p}\cdot\boldsymbol{\xi}) - \frac{(\mathbf{p}\cdot\boldsymbol{\xi})(\mathbf{p}\cdot\mathbf{q})}{E_\mu + m_\mu}\right\}.
\end{aligned}
\tag{3.12}
$$

This expression has been derived earlier by Gatto[4]) in a slightly different form.

If no polarization measurements are carried out we have to integrate over all possible directions of polarization, and terms linear in $\boldsymbol{\xi}$ vanish. From (3.11) one obtains after a few simplifications

$$
|M|^2 \propto 2f[fp^\sigma k_\sigma + m_\mu^2 g]k^\lambda q_\lambda + [m_\mu^2 g^2 - m_K^2 f^2]p^\lambda q_\lambda. \tag{3.13}
$$

If we are interested only in the correlation between muon and K-meson momenta eq. (3.13) gives the correlation function

$$
\Gamma(\mu, k) = 1 - \frac{2f^2 E_K}{2f^2 E_\mu E_K^2 + 2m_\mu^2 fgE_K + m_\mu^2 g^2 E_\mu - m_K^2 f^2 E_\mu}\,\mathbf{p}\cdot\mathbf{k},
$$

or written in a slightly different form

$$
\Gamma(\mu, k) = 1 - \frac{2}{2 - \kappa_1^2 + \dfrac{2\kappa_1\kappa_2}{\gamma} + \dfrac{\kappa_1^2}{\gamma^2}}\,\frac{v_\mu v_K}{c^2}\cos(\mathbf{p}, \mathbf{k}) \tag{3.14}
$$

with

$$\gamma = \frac{m_K f}{m_\mu g}, \quad \kappa_1 = \frac{m_K}{E_K}, \quad \kappa_2 = \frac{m_\mu}{E_\mu}.$$

From arguments based on perturbation theory it follows that the energy dependence of the functions f and g can be neglected, and comparison between experimental results and theory gives the following possible values for the parameter γ [7]):

$$\gamma = -0.95, \quad \gamma = 8.3.$$

A particularly simple correlation function for the muon and the antineutrino momenta is obtained in the case of a K-meson decaying at rest. From (3.13) one easily finds the correlation function

$$\Gamma(\mu, \nu) = 1 + \frac{\gamma^2 - 1}{1 + \gamma^2 + 2\gamma\kappa} \frac{v}{c} \cos(\mathbf{p}, \mathbf{q}), \tag{3.15}$$

where γ denotes the same quantity as before, v is the velocity of the muon, and $\kappa = m_\mu / E_\mu$.

The case of a K-meson decaying at rest with no polarization measurements has been discussed by Zachariasen [4]). This case can be obtained from (3.12) by integration over possible directions of polarization. Then terms linear in ξ vanish and the square of the matrix element can be written

$$|M|^2 \propto m_K^2 f^2 [E_\mu E_\nu + \mathbf{p} \cdot \mathbf{q}] + 2 m_\mu^2 m_K f g E_\nu + m_\mu^2 g^2 [E_\mu E_\nu - \mathbf{p} \cdot \mathbf{q}]. \tag{3.16}$$

Making use of energy-momentum conservation

$$m_K = E_\pi + E_\mu + E_\nu, \quad \boldsymbol{\pi} + \mathbf{p} + \mathbf{q} = 0,$$

we arrive at

$$\mathbf{p} \cdot \mathbf{q} = \tfrac{1}{2} [2 E_\mu E_\nu - m_K^2 - m_\pi^2 + m_\mu^2 + 2 m_K E_\pi]$$

and (3.16) can be written

$$\begin{aligned} |M|^2 \propto{}& \tfrac{1}{2} m_K^2 f^2 [4 E_\mu E_\nu - m_K^2 - m_\pi^2 + m_\mu^2 + 2 m_K E_\pi] \\ &+ 2 m_\mu^2 m_K f g \, E_\nu + \tfrac{1}{2} m_\mu^2 g^2 [m_K^2 + m_\pi^2 - m_\mu^2 - 2 m_K E_\pi], \end{aligned} \tag{3.17}$$

which is the result given by Zachariasen, except for the sign of the second term. This difference is due to the fact that Zachariasen has treated the K^+-decay instead of the K^--decay discussed here.

The calculations are carried through for the $K_{\mu 3}$ mode. As the muon and the electron are identical particles except for the mass difference, the same expression for the matrix element is obtained in the case of the K_{e3} mode. In particular it should be noted that the unknown functions are the same for both modes.

4. The Polarization State for the Outgoing Muon (Electron)

It has already been mentioned that the three-body K-meson decay is of particular interest as far as the polarization of the outgoing muon is concerned. This is the case because these two processes are the only ones known where no angular momentum selection rules impose special restrictions.

In the V—A theory only the chiral projection $\frac{1}{2}(1+\gamma_5)\psi$ 'exists'. From this it directly follows that for every particle with vanishing rest mass the spin vector is antiparallel to the momentum vector of the particle. For antiparticles with vanishing rest mass the spin vector points in the same direction as the momentum.

In the simplest case of a K-meson decaying at rest the polarization vector for the outgoing muon (electron) is most easily given by the projections along the momentum vectors of muon (electron) and the antineutrino. If we denote these projections by $P^{(\mu)}$ and $P^{(\nu)}$ one obtains from eq. (3.12)

$$
\begin{aligned}
P^{(\mu)} &= \frac{p}{M_0(E_\mu+m_\mu)}\{-m_K{}^2f^2[E_\mu E_\nu+m_\mu E_\nu+(\mathbf{p}\cdot\mathbf{q})] \\
&\quad +2m_\mu m_K fg(\mathbf{p}\cdot\mathbf{q})+m_\mu{}^2g^2[E_\mu E_\nu+m_\mu E_\nu-(\mathbf{p}\cdot\mathbf{q})]\}, \\
P^{(\nu)} &= -\frac{m_\mu q}{M_0}[m_K{}^2f^2+2m_K E_\mu fg+m_\mu{}^2g^2], \qquad (4.1) \\
M_0 &= m_K{}^2f^2[E_\mu E_\nu+(\mathbf{p}\cdot\mathbf{q})]+2m_K m_\mu fgm_\mu E_\nu+m_\mu{}^2g^2[E_\mu E_\nu-(\mathbf{p}\cdot\mathbf{q})].
\end{aligned}
$$

In the limiting case $m_\mu = 0$ one obtains from (4.1)

$$P^{(\mu)} = -1, \quad P^{(\nu)} = 0,$$

as one would expect from the fact that only the positive chiral projection exists. This limiting case is a good approximation for the electron mode, but it is too rough for the $K_{\mu 3}$-mode. In the latter case the non-vanishing mass of the muon gives rise to a component along the antineutrino momentum besides the longitudinal component. To investigate the polarization in this case more detailed information about the unknown functions f and g is required. This information can in principle be obtained from the experimental material, as proposed, for example, by Gatto [4]). Owing to the poor statistics, however, no definite conclusions can be drawn, and to get further in the analysis one is forced to assume energy independence for the functions f and g as a reasonable approximation. This means that the parameter earlier denoted by γ is a constant which can be determined from the comparatively simple distribution functions (3.14) or (3.15). When this parameter γ is determined the polarization is uniquely predicted by (4.1).

5. Conclusions

The polarization state of the outgoing muon (electron) in three-body K-meson decay is uniquely determined by a parameter γ, which also appears in a simple way in the correlation function between the muon and K-meson momenta or between muon and antineutrino momenta for a K-meson decaying at rest. This parameter seems to have an energy dependence that can be neglected and consequently the polarization is completely determined by a constant. The numerical value of this constant can be determined by investigation of one of the correlation functions mentioned above.

The author is indebted to Professor R. E. Marshak and Dr. E. C. G. Sudarshan for suggesting this problem and for many helpful discussions during the author's stay at the University of Rochester as a Rotary Foundation Fellow. He also wants to express his sincere gratitude to Professor N. Svartholm for valuable help and criticism.

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PSEUDO-CLUSTER EXPANSION

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Abstract: The Ursell-Mayer cluster expansion method is extended to the case of a q-number model operator for either boson or fermion systems. The many-body problem for a system with strong, hard sphere interactions can be approximately reduced to that for a weakly interacting system without hard sphere interactions. The effective Hamiltonian for the weakly interacting system is given in the form of an expansion series quite analogous to the ordinary cluster expansion. (We call it the "pseudo-cluster expansion" in this paper.) The validity of our expansion method is discussed. It can be proved that in the simplest case we are led to the approximate Bethe-Goldstone equation (or Brueckner approximation). We derive formulas expressing the total energy and the diagonal and non-diagonal matrix elements of operators by the asymptotic wave functions. Possibilities of application of our method are discussed.

1. Introduction

Recent studies of the nuclear saturation problem have made remarkable progress. They are grossly classified into two types: a) Brueckner's approximation [1, 2, 3]) and b) the variational treatment using the cluster expansion method [4, 5, 6]). The latter treatment has been discussed only under very simplified assumptions with respect to the two-body nucleon correlations. The most convenient form of the correlation function is the so-called Jastrow type:

$$\Psi = M\Phi, \tag{1}$$

with

$$M = \prod_{(i,j)} f(r_{ij}), \tag{2}$$

where Ψ is the true wave function of the ground state satisfying the Schroedinger equation

$$H\Psi = E\Psi \tag{3}$$

and the model operator of the Jastrow type as in (2) is assumed to have the following properties:

$$f(r_{ij}) = \begin{matrix} 0 & \text{for} & r_{ij} = D \\ 1 & \text{for} & r_{ij} \to \infty, \end{matrix} \tag{4}$$

where r_{ij} represents the distance between the positions of i-th and j-th particles

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and D the radius of a "hard sphere". Most previous treatments seem to be based on the idea that we know the model wave function Φ well and the main task is to determine the structure of the model operator M by the Schroedinger equation (3). According to the variation method we assume the Fermi gas model or shell model for Φ and minimize the expectation value of the total Hamiltonian H by changing the parameters contained in the correlation function $f(r)$.

On the other hand, the Brueckner approximation method has been formulated apparently in an entirely different way. However, we can also understand it in such a way that for a fixed Φ we determine the parts of the velocity-dependent model operator M which are mostly responsible for the total energy E, by solving the Bethe-Goldstone equation instead of solving the Schroedinger equation (3) rigorously †.

The aim of this paper is to propose a new method to treat a strongly interacting system satisfying suitable conditions to be stated later. Both treatments a) and b) can be regarded as special cases and it is very easy to understand the relation between them. Roughly speaking, our main idea is to isolate in the form (1) a velocity-dependent short range correlation part: here "short range" correlation means a correlation compatible with the idea of cluster expansion. We shall derive the effective Schroedinger equation which the asymptotic wave function Φ should satisfy:

$$H_0\Phi = E\Phi. \tag{5}$$

In this paper we would call the method to derive the effective Hamiltonian H_0 from the true Hamiltonian H the "pseudo-cluster expansion" method. The essence of our method is probably quite similar to the "nearest-neighbour expansion" of Riesenfeld-Watson [3]), though the apparent features are entirely different.

In sec. 2 the method is discussed in detail. Our starting equations are eqs. (1), (3) and (5). The formalism of sec. 2 is of sufficiently general character, but the intuitive Jastrow case will be exemplified everywhere.

If we can solve the effective Schroedinger equation (5) by some means exactly, the eigenvalues E are the same as the true eigenvalues, and the true eigenfunctions Ψ can be obtained by the relation (1). The advantage of treating (5) rather than (3) is the following:

1) The model Hamiltonian H_0 does not contain any hard core interaction, so that we have several ways of solving the differential equation, if we want to make the effort. In fact the forces involved in H_0 are quite weak compared with the original ones. This suggests the appearance of shell aspects in the effective

† The equation (3) leads to $E = \dfrac{(\phi, H\Psi)}{(\phi, \Psi)} = \dfrac{(H\phi, \Psi)}{(\phi, \Psi)}$ for any function ϕ, so that E can be determined by investigating only some properties of the true wave function Ψ.

many-body problem (5). Many excellent theories using the Fourier components of the interaction can be applied to (5), but not to (3).

2) In the many-body problem generally, there should exist both long and short range correlations in the wave function. If the particles can be regarded as approximately free, although there exists some kind of long range correlation †, our method is useful for treating such systems. It is possible in principle to discuss which of the shell model or the collective model is more favourable in some given nucleus.

On the other hand we meet some disadvantages in treating the effective many-body problem:

1) If the operator M were a unitary operator, the orthonormal complete set Ψ_i would correspond to another orthonormal complete set Φ_i by the relation $\Psi_i = M\Phi_i$. But this is not the case generally (see fig. 1). Accordingly it is

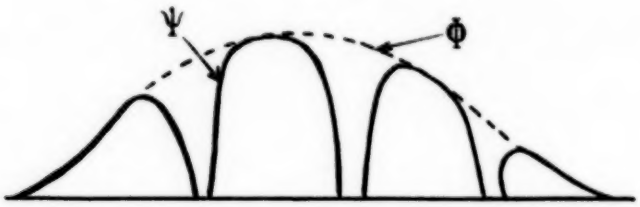

Fig. 1. Definition of the asymptotic wave function Φ (eq. (5)). The true wave functions Ψ are orthonormal, but the asymptotic wave functions Φ are not so rigorously in general.

necessary in quantitative discussions to take into account the effect of the small deviation from the orthonormality. A rigorous relation is resembling the orthogonality relation of the Hermite polynominals

$$(\Psi_i, \Psi_j) = (M\Phi_i, M\Phi_j) = 0 \quad (i \neq j). \tag{6}$$

However, it is shown later that the Φ's are approximately orthogonal. If we can choose the model operator M such that

$$[J, M] = 0, \quad [\Pi, M] = 0, \tag{7}$$

where the J and Π represent the total angular momentum and parity operators, the Ψ's having different spins or parities correspond to exactly orthogonal asymptotic wave functions Φ's.

2) As shown in this paper, the effective Schroedinger equation (5) is a complicated integro-differential equation in the higher cluster terms. We must solve the equation for instance by the method of successive approximations.

In order to choose a suitable M the simplest method is to take into account only the second order cluster terms. It is shown in sec. 3 that to this order the

† A system of high viscidity cannot be treated by our method. However, the particles in a nucleus should be approximately free, because it has clearly the shell model aspects. Bohr model etc. do not contradict the approximate freedom of particle motion.

equation effectively corresponds to the well-known Bethe-Goldstone equation, so that this approximation corresponds to the Brueckner approximation.

2. Pseudo-cluster Expansion

We choose any operator M in eq. (1) which has the following properties [6]):

(I) The product property †

$$M(1, 2, \ldots, N) = M^{(n)}(1, 2, \ldots, n)M^{(N-n)}(n+1, \ldots, N), \tag{8}$$

provided that

$$r_{ij} \equiv |\mathbf{x}_i - \mathbf{x}_j| > \delta \quad \text{for every} \begin{cases} i = 1, 2, \ldots, n \\ j = n+1, \ldots, N; \end{cases}$$

δ stands for the "healing distance" [2]). By this equation the (generally q-number) operators $M^{(2)}, M^{(3)}, \ldots M^{(N-1)}$ can be defined, if M is given. In the formulation below it is not necessary at all to assume that the $M^{(n)}$'s are functions only of the space coordinates of the particles.

(II) Normalization for $M^{(1)}$:

$$M^{(1)}(i) = 1. \tag{9}$$

Since $M = 1$ if $r_{ij} > \delta$ for every $i, j = 1, \ldots, N$, the wave function Ψ deviates from the asymptotic wave function when any two particles approach each other within the distance δ.

(III) The only essential condition imposed on the operator M and the corresponding Φ simultaneously is the convergence of our pseudo-cluster expansion: "The pseudo-cluster terms defined below are not of a larger order of magnitudes than the ordinary cluster terms." Strictly speaking, it is not clear whether the condition (III) is satisfied or not until we check the validity of (III) for the M and the exact Φ which can be obtained by solving (5).

For convenience we assume that

$$(\Phi, \Phi) = 1. \tag{10}$$

The original Hamiltonian H is written as

$$\begin{aligned} H \equiv H^{(1)} + H^{(2)} &= \sum_k H_k^{(1)} + \sum_{(ij)} H_{ij}^{(2)} \\ &= -\frac{\hbar^2}{2M} \sum_k \Delta_k + \sum_{(ij)} V_{ij}. \end{aligned} \tag{11}$$

The expectation value E can be expressed as

$$E = \left[\frac{\partial}{\partial \beta} \log I_\beta^{(N)} \right]_{\beta=0}, \tag{12}$$

† The expression on the right hand side of eq. (8) is consistent with the symmetry property o. the left hand side, because we restrict ourselves to some specific region of configuration spacef

where

$$I_\beta^{(N)} = (\Phi, M^+ e^{\beta(H^{(1)}+H^{(2)})} M\Phi). \tag{13}$$

Now, quite analogously to the ordinary cluster expansion method, we define the pseudo-cluster integrals $x_\beta^{(i)}$ successively:

$$I_\beta^{(1)} \equiv (\Phi, M_i^{(1)+} e^{\beta H_i^{(1)}} M_i^{(1)}\Phi) = 1+O(\beta), \tag{14a}$$

$$I_\beta^{(2)} \equiv (\Phi, M_{ij}^{(2)+} e^{\beta H_{ij}^{(2)}+\beta(H_i^{(1)}+H_j^{(1)})} M_{ij}^{(2)}\Phi) = (\Phi, e^{\beta(H_i^{(1)}+H_j^{(1)})}\Phi)(1+x_\beta^{(2)}), \tag{14b}$$

$$\begin{aligned} I_\beta^{(3)} &\equiv (\Phi, M_{ijk}^{(3)+} e^{\beta(H_{ij}^{(2)}+H_{jk}^{(2)}+H_{ik}^{(2)})+\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)})} M_{ijk}^{(3)}\Phi) \\ &= (\Phi, e^{\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)})}\Phi)(1+{}_3C_2 x_\beta^{(2)}+x_\beta^{(3)}), \end{aligned} \tag{14c}$$

$$\begin{aligned} I_\beta^{(4)} &\equiv (\Phi, M_{ijkl}^{(4)+} e^{\beta(H_{ij}^{(2)}+H_{ik}^{(2)}+H_{il}^{(2)}+H_{jl}^{(2)}+H_{jk}^{(2)}+H_{kl}^{(2)})+\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)})} M_{ijkl}^{(4)}\Phi) \\ &= (\Phi, e^{\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)})}\Phi)(1+{}_4C_2 x_\beta^{(2)}+{}_4C_3 x_\beta^{(3)}+x_\beta^{(4)'}) \qquad (14d) \\ &= (\Phi, e^{\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)})}\Phi)(1+{}_4C_2 x_\beta^{(2)}+{}_4C_3 x_\beta^{(3)}+\tfrac{1}{2}{}_4C_2{}_2C_2 x_\beta^{(2)2}+x_\beta^{(4)}), \end{aligned}$$

$$\begin{aligned} I_\beta^{(N)} &\equiv (\Phi, M^+ e^{\beta H^{(2)}+\beta H^{(1)}} M\Phi) \\ &= (\Phi, e^{\beta H^{(1)}}\Phi)(1+{}_NC_2 x_\beta^{(2)}+{}_NC_3 x_\beta^{(3)}+{}_NC_4 x_\beta^{(4)}+\ldots+{}_NC_N x_\beta^{(4)}) \\ &= (\Phi, e^{\beta H^{(1)}}\Phi)\Big\{1+{}_NC_2 x_\beta^{(2)}+{}_NC_3 x_\beta^{(3)}+\frac{1}{2!}{}_NC_2{}^2 x_\beta^{(2)2} \\ &\qquad -\frac{1}{2!}({}_NC_2{}^2-{}_NC_4{}_4C_2{}_2C_2) x_\beta^{(2)2}+{}_NC_4 x_\beta^{(4)}+\ldots\Big\} \\ &\approx (\Phi, e^{\beta H^{(1)}}\Phi)\exp N\Big\{\frac{1}{N}{}_NC_2 x_\beta^{(2)}+\frac{1}{N}{}_NC_3 x_\beta^{(3)} \\ &\qquad -\frac{1}{2!N}({}_NC_2{}^2-{}_NC_4{}_4C_2{}_2C_2) x_\beta^{(2)2}+\frac{{}_NC_4}{N} x_\beta^{(4)}+\ldots\Big\}. \end{aligned} \tag{14e}$$

The equations (14b), (14c), (14d), ... (14e) give the definitions of $x_\beta^{(2)}$, $x_\beta^{(3)}$, $x_\beta^{(4)'}$, $x_\beta^{(4)}$, ... $x_\beta^{(N)}$ successively. The $x_\beta^{(N)}$'s do not depend on the specific numbers of particles, provided that Φ is symmetrized or antisymmetrized with respect to all the numbers. For the systematic derivation of the last step in (14e) we can refer to the paper by Tolhoek *et al.* [6]). If we choose plane waves in place of Φ, the series of $x_\beta^{(n)}$ in the exponent of (14e) just corresponds to the ordinary cluster expansion series. The only difference between the ordinary and our pseudo-cluster integrals is that each cluster integral implicitly involves suitable higher cluster terms in Φ.

By solving the equations (14) we obtain

$$x_\beta^{(2)} = \frac{I_\beta^{(2)}}{(\Phi,\, e^{\beta(H_i^{(1)}+H_j^{(1)})}\Phi)} - 1$$

$$x_\beta^{(3)} = \frac{I_\beta^{(3)}}{(\Phi,\, e^{\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)})}\Phi)} - {}_3C_2\, x_\beta^{(2)} \tag{15}$$

$$x_\beta^{(4)} = \frac{I_\beta^{(4)}}{(\Phi,\, e^{\beta(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)})}\Phi)} - {}_4C_2\, x_\beta^{(2)} - {}_4C_3\, x_\beta^{(3)} - \tfrac{1}{2}{}_4C_{2\,2}C_2\, x_\beta^{(2)2}.$$

Now we are at the stage of discussing the convergence of the series in the exponent of (14e). The necessary condition for the convergence of the ordinary cluster expansion has already been discussed by many authors [4, 5, 6]. As an illustration let us choose the Jastrow type operator M (eq. (2)). Then, the explicit forms for $x_0^{(2)}$, $x_0^{(3)}$, ... are given by

$$x_0^{(2)} = (\Phi,\, h(r_{ij})\Phi), \tag{16a}$$

$$x_0^{(3)} = 3(\Phi,\, h(r_{ij})h(r_{jk})\Phi) + (\Phi,\, h(r_{ij})h(r_{jk})h(r_{ki})\Phi), \tag{16b}$$

$$\begin{aligned} x_0^{(4)} = {} & 3\{(\Phi,\, h(r_{ij})h(r_{kl})\Phi) - (\Phi,\, h(r_{ij})\Phi)^2\} \\ & + 4\{(\Phi,\, h(r_{ij})h(r_{ik})h(r_{il})\Phi) + 8(\Phi,\, h(r_{ij})h(r_{jk})h(r_{kl})\Phi) \\ & + 3(\Phi,\, h(r_{ij})h(r_{jk})h(r_{kl})h(r_{li})\Phi) \\ & + 6(\Phi,\, h(r_{ij})h(r_{jk})h(r_{kl})h(r_{li})h(r_{ik})\Phi) \\ & + (\Phi,\, h(r_{ij})h(r_{jk})h(r_{kl})h(r_{li})h(r_{ik})h(r_{jl})\Phi)\}, \end{aligned} \tag{16c}$$

where

$$f^2(r_{ij}) = 1 + h(r_{ij}). \tag{16d}$$

In order to estimate them we put

$$\left| \int h(r) 4\pi r^2 \mathrm{d}r \right| = \omega. \tag{17}$$

Therefore, if the Φ's represent plane waves,

$$x_0^{(2)} = \mathrm{O}\left(\frac{\omega}{\Omega}\right), \quad x_0^{(3)} = \mathrm{O}\left(\frac{\omega^2}{\Omega^2}\right), \quad x_0^{(4)} = \mathrm{O}\left(\frac{1}{N}\frac{\omega^2}{\Omega^2}\right) + \mathrm{O}\left(\frac{\omega^3}{\Omega^3}\right), \tag{18}$$

where Ω means the total volume of the system. It is to be noticed that the first curly bracket on the right hand of (16c) has an order of magnitude with an additional factor $1/N$. This fact comes from the well-known product property of the plane wave functions. In Jastrow's case it is clear that the necessary condition for the convergence of the series can be expressed as

$$N\frac{\omega}{\Omega} \ll 1. \tag{19}$$

The condition (19) means that the volume where $h(r)$ differs appreciably from zero should be much smaller than the average volume occupied by one particle, so that the shape of $h(r)$ should be of sufficiently short range.

It should be stressed that the above order of magnitude relations (18) can be valid even if the Φ's are not exactly plane waves. For instance, they are also valid in the case in which there exist over-all correlations, but the particles are approximately free. Although Φ contains the residual correlations, the relations continue to be valid, in so far as Φ has a product property similar to eq. (8) †. Of course it is not the case if Φ is like a liquid of high viscidity. The condition III is satisfied if the relation (18) is still valid for the true effective wave function Φ in place of plane waves, though the equations (16)'s are not adopted generally.

Let us proceed with our discussion by assuming that the relations (18) are satisfied by the true asymptotic wave functions Φ. A plausible argument for the required product property of Φ will be given later.

Under these assumptions, eqs. (12) and (14e) lead to the pseude-cluster expansion for the energy expectation value ††

$$E = E^{(0)}+E^{(1)}+E^{(2)}+\ldots, \tag{20a}$$

with

$$E^{(0)} = (\Phi, H^{(1)}\Phi), \tag{20b}$$

$$\begin{aligned} E^{(1)} = {}_NC_2\{&(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi) \\ &-(\Phi, (H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}, \end{aligned} \tag{20c}$$

and

$$\begin{aligned} E^{(2)} = {}&{}_NC_3[\{(\Phi, M_{ijk}^{(3)+}(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_{ij}^{(2)}+H_{jk}^{(2)}+H_{ki}^{(2)})M_{ijk}^{(3)}\Phi) \\ &-(\Phi, (H_i^{(1)}+H_j^{(1)}+H_k^{(1)})\Phi)(\Phi, M_{ijk}^{(3)+}M_{ijk}^{(3)}\Phi)\} \\ &-{}_3C_2\{(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi) \\ &-(\Phi, (H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}] \\ &-({}_NC_2{}^2-{}_NC_4{}_4C_2{}_2C_2)\{(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)-1\} \\ &\times\{(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi)-(\Phi, (H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\} \\ &+{}_NC_4[\{(\Phi, M_{ijkl}^{(4)+}(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)} \\ &\quad +H_{ij}^{(2)}+H_{ik}^{(2)}+H_{il}^{(2)}+H_{jk}^{(2)}+H_{jl}^{(2)}+H_{kl}^{(2)})M_{ijkl}^{(4)}\Phi) \\ &\quad -(\Phi, (H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_l^{(1)})\Phi)(\Phi, M_{ijkl}^{(4)+}M_{ijkl}^{(4)}\Phi)\} \end{aligned} \tag{20d}$$

† Instead of δ in eq. (8), $|\Phi|^2$ may have another healing distance δ' ($\delta' > \delta$). It is essential for the validity of (18) that

$$\frac{(\delta')^3}{\Omega} = \mathrm{O}\left(\frac{1}{N}\right).$$

†† We can easily verify that each term on the right hand side of eqs. (20) just corresponds to each term of the eqs. (31) and (32) of ref. [5]) if we choose for Φ the Slater determinant of the one-particle wave functions.

$$
\begin{aligned}
&-{}_4C_2\{(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi)-(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}\\
&-{}_4C_3[\{(\Phi, M_{ijk}^{(3)+}(H_i^{(1)}+H_j^{(1)}+H_k^{(1)}+H_{ij}^{(2)}+H_{jk}^{(2)}+H_{ki}^{(2)})M_{ijk}^{(3)}\Phi)\\
&\qquad -(\Phi,(H_i^{(1)}+H_j^{(1)}+H_k^{(1)})\Phi)(\Phi, M_{ijk}^{(3)+}M_{ijk}^{(3)}\Phi)\}\\
&-{}_3C_2\{(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi)-(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}\\
&-{}_4C_2\{(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)-1\}\{(\Phi, M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}\Phi)\\
&\qquad -(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}]+\ldots..
\end{aligned}
\tag{20d}
$$

3. The Effective Hamiltonian H_0

If we minimize E for fixed M by varying Φ, we obtain the pseudo-cluster expansion of the effective Schroedinger equation (5) for the asymptotic wave function Φ

$$(H_0-E)\Phi=0 \quad \text{and} \quad (\Phi,\Phi)=1, \tag{5'}$$

where

$$H_0=H_0^{(0)}+H_0^{(1)}+H_0^{(2)}+\ldots, \tag{21a}$$

$$H_0^{(0)}=\sum_k H_k^{(1)}, \tag{21b}$$

$$
\begin{aligned}
H_0^{(1)} &= \sum_{(ij)}\{M_{ij}^{(2)+}(H_i^{(1)}+H_j^{(1)}+H_{ij}^{(2)})M_{ij}^{(2)}-M_{ij}^{(2)+}M_{ij}^{(2)}(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)\\
&\quad -(H_i^{(1)}+H_j^{(1)})(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)+(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)\}\\
&= \sum_{(ij)}\{M_{ij}^{(2)+}[H_i^{(1)}+H_j^{(1)}, M_{ij}^{(2)}]+M_{ij}^{(2)+}H_{ij}^{(2)}M_{ij}^{(2)}\}\\
&\quad +\sum_{(ij)}\{M_{ij}^{(2)+}M_{ij}^{(2)}-(\Phi, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi)\}\{H_i^{(1)}+H_j^{(1)}-(\Phi,(H_i^{(1)}+H_j^{(1)})\Phi)\}+\ldots..
\end{aligned}
\tag{21c}
$$

Similarly, we can easily derive the higher cluster terms from eq. (20), but we shall not write them down explicitly here.

Eq. (5') must be solved under the boundary conditions

$$\Phi=\begin{cases}0\\ \text{finite}\end{cases} \quad \text{for} \quad \begin{matrix}|\mathbf{x}_i|=\infty\\ r_{ij}=D.\end{matrix}$$

If we can obtain the solution satisfying the above conditions, Ψ automatically satisfies the boundary conditions

$$\Psi=0 \quad \text{for} \quad |\mathbf{x}_i|=\infty \quad \text{and} \quad r_{ij}=D.$$

It can be noticed that $H_{ij}^{(2)}$ appears on the right hand side of (21) always accompanied by $M_{ij}^{(2)}$. Thus, we can choose M in such a manner that the effective Hamiltonian H_0 has no hard core interaction.

Let us consider the Jastrow case again. Eq. (21) becomes

$$H_0 = \sum_k H_k^{(1)} + \sum_{ij} f(r_{ij}) V_{ij} f(r_{ij})$$
$$- \frac{\hbar^2}{M} \sum_{(ij)} \{ f(r_{ij}) \Delta_i f(r_{ij}) + f(r_{ij}) \nabla_i f(r_{ij}) \cdot \nabla_i \} \qquad (22)$$
$$+ \sum_{(ij)} \{ h(r_{ij}) - (\Phi, h(r_{ij}) \Phi) \} \{ H_i^{(1)} + H_j^{(1)} - (\Phi, (H_i^{(1)} + H_j^{(1)}) \Phi) \} + \ldots .$$

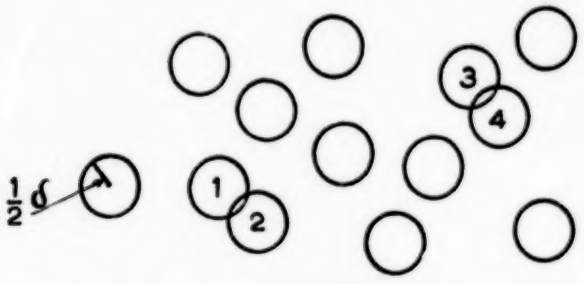

Fig. 2. Schematic picture of the reducible 4-th cluster term (eq. (14d)). δ stands for the healing distance. The validity of cluster expansion is due to the weakness of the coupling between the pairs (1, 2) and (3,4).

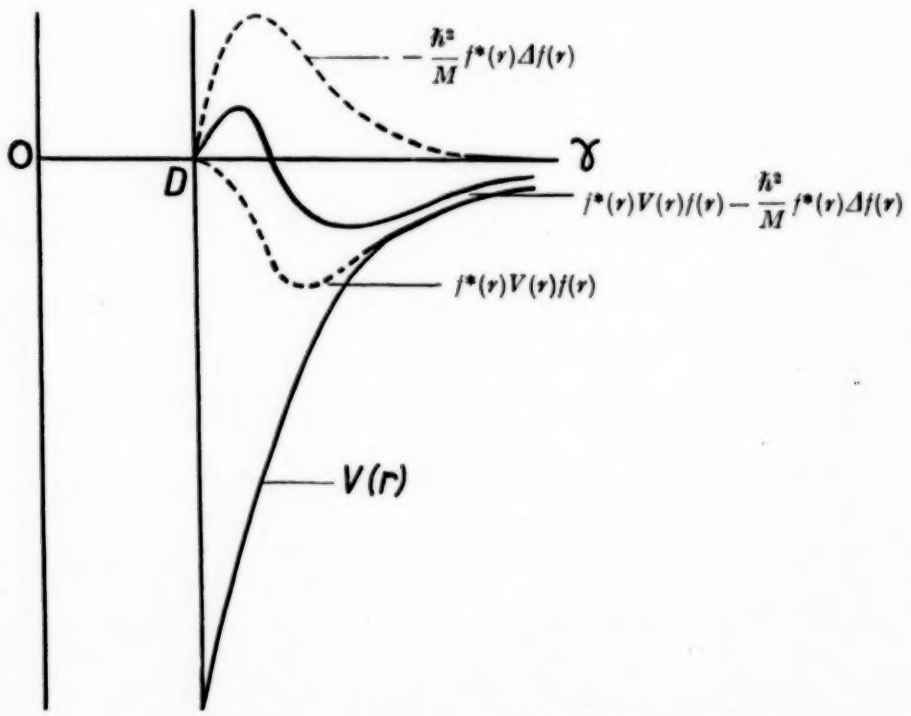

Fig. 3. Effective potential in Jastrow's case (eq. (22)). See Iwamoto-Yamada (ref. [5]), p. 352).

The first three terms of the effective Hamiltonian H_0 consist of the ordinary kinetic energy T, the screened potential energy $f(r)Vf(r)$ and the apparent (repulsive) potential resulting from the curvature of the correlation function $f(r)$. Therefore, it is clear in this case that H_0 does not contain any hard core inter-

action and the effective potentials in H_0 are greatly reduced compared with the original interaction V.

So far we have not adopted any special form for M. Provided that it satisfies the conditions (I) to (III), M can be chosen arbitrarily. However, it is practically impossible to handle the complicated higher cluster terms. The practically best way of choosing $M^{(2)}$ is the minimization of the approximate expression for the energy, all the higher cluster terms $H_0^{(2)}$, $H_0^{(3)}$, . . . being cut off. If the $M^{(2)}$ so obtained satisfies the condition (19) we adopt it and then solve the eq. (5′).

Now let us vary the $M^{(2)}$'s in $E^{(1)}+E^{(2)}$ of eq. (20) for a fixed Φ:

$$\sum_{(ij)}\{H_i^{(1)}+H_j^{(1)}-\frac{2}{N}E+H_{ij}^{(2)}\}M_{ij}^{(2)}\Phi=0. \tag{23}$$

Eq. (23) is quite similar to the ordinary wave equation governing the scattering of a pair of particles, since $M^{(2)}\to 1$ for $r_{ij}\to\infty$.

Let us assume that Φ is a Slater determinant of plane waves, which describes a fermion assembly:

$$\Phi=\sum\varepsilon_P\prod\phi_{i_P}(x_i) \tag{24}$$

where x_{i_P} stands for the space, spin and isospin coordinates of i_P-th particle. We get from (23) and (24)

$$\sum_{(ij)}\sum_P\varepsilon_P\left\{H_i^{(1)}+H_j^{(1)}-\frac{2}{N}E+H_{ij}^{(2)}\right\}M_{ij}^{(2)}\phi_{i_P}(x_i)\phi_{j_P}(x_j)\prod_{k\neq i,j}{}'\phi_{k_P}(x_k)=0. \tag{25}$$

Taking into account the exclusion principle,

$$\sum_{(ij)}\sum_P\varepsilon_P\,Q_{i_Pj_P}\left\{\left(H_i^{(1)}+H_j^{(1)}-\frac{2}{N}E+H_{ij}^{(2)}\right)M_{ij}^{(2)}\phi_{i_P}(x_i)\phi_{j_P}(x_j)\right\}\prod_{k\neq i,j}{}'\phi_{k_P}(x_k)=0, \tag{26}$$

where $Q_{i_Pj_P}$ is the projection operator [2]) which omits the Fourier component corresponding to the occupied levels except i_P and j_P. If we put

$$\begin{aligned}\psi_{\alpha\beta}(x_i,x_j)&\equiv M_{ij}^{(2)}\{\phi_\alpha(x_i)\phi_\beta(x_j)-\phi_\beta(x_i)\phi_\alpha(x_j)\}\\&=f_{\alpha\beta}(x_ix_j)\{\phi_\alpha(x_i)\phi_\beta(x_j)-\phi_\beta(x_i)\phi_\alpha(x_j)\},\end{aligned} \tag{27}$$

the solution of the Bethe-Goldstone equation †

$$(H_i^{(1)}+H_j^{(1)}-\varepsilon_{\alpha\beta})\psi_{\alpha\beta}(x_ix_j)=-Q_{\alpha\beta}H_{ij}^{(2)}\psi_{\alpha\beta}(x_ix_j) \tag{28}$$

† The equation

$$Q\{H_i^{(1)}+H_j^{(1)}-\varepsilon_{\alpha\beta}+H_{ij}^{(2)}\}\psi_{\alpha\beta}=0$$

is equivalent to

$$(H_i^{(1)}+H_j^{(1)}-\varepsilon_{\alpha\beta})\psi_{\alpha\beta}+QH_{ij}^{(2)}\psi_{\alpha\beta}=0$$

if the ϕ's are eigenfunctions of $H^{(1)}$.

with

$$\sum_{(\alpha\beta)} \varepsilon_{\alpha\beta} = (N-1)E$$

roughly satisfies the equation (26), since on the average $\varepsilon_{\alpha\beta} \approx (2/N)E$.

Therefore, it is practically the best way to choose $M^{(2)}$ such that the $f_{\alpha\beta}$ defined by the relation (27) satisfy the Bethe-Goldstone equation.

4. Pseudo-Cluster Expansion of Matrix Elements

Let us assume that the solutions of the effective Schroedinger equation (5′) could be obtained and let a set of solutions be $\Phi_0, \Phi_1, \ldots$. Now, we will consider a matrix element of the operator $\mathscr{M}$ between the states Ψ_m and Ψ_n:

$$\mathscr{M}_{mn} = \frac{(\Psi_m, (\mathscr{M}^{(1)}+\mathscr{M}^{(2)})\Psi_n)}{\sqrt{(\Psi_m, \Psi_m)(\Psi_n, \Psi_n)}}, \tag{27}$$

where $\mathscr{M} = \mathscr{M}^{(1)}+\mathscr{M}^{(2)} = \sum_k \mathscr{M}_k^{(1)} + \sum_{ij} \mathscr{M}_{ij}^{(2)}$ stands for the sum of one-body and two-body operators. We may write

$$\begin{aligned}\mathscr{M}_{mn} &= \frac{(\Phi_m, M^+(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})M\Phi_n)}{\sqrt{(\Phi_m, M^+M\Phi_m)(\Phi_m, M^+M\Phi_n)}} \\ &= \sqrt{\frac{(\Phi_m, M^+(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})M\Phi_n)}{(\Phi_m, M^+M\Phi_m)}} \times \sqrt{\frac{(\Phi_m, M^+(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})M\Phi_n)}{(\Phi_n, M^+M\Phi_n)}}.\end{aligned} \tag{28}$$

By the same prescription as in section 2, we can obtain the pseudo-cluster expansion series of each quantity under the square root in the right hand side of (28):

$$\begin{aligned}&\frac{(\Phi_m, M^+(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})M\Phi_n)}{(\Phi_m, M^+M\Phi_m)} = \left[\frac{\partial^2}{\partial\beta\partial\gamma}\log\,(\Phi_m, M^+ e^{\beta(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})} M e^{\gamma\frac{\Phi_n}{\Phi_m}\Phi_m})\right]_{\beta=\gamma=0} \\ &+\left[\frac{\partial}{\partial\beta}\log(\Phi_m, M^+ e^{\beta(\mathscr{M}^{(1)}+\mathscr{M}^{(2)})} M\Phi_m)\right]_{\beta=0}\left[\frac{\partial}{\partial\gamma}\log(\Phi_m, M^+ M e^{\gamma\frac{\Phi_n}{\Phi_m}\Phi_m})\right]_{\gamma=0} \\ &= (\Phi_m, \mathscr{M}^{(1)}\Phi_n) - (\Phi_m, \mathscr{M}^{(1)}\Phi_m)(\Phi_m, \Phi_n) \\ &-2\sum_{(ij)}(\Phi_m, \Phi_n)\{(\Phi_m, M_{ij}^{(2)+}(\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)}+\mathscr{M}_{ij}^{(2)})M_{ij}^{(2)}\Phi_m) \\ &\qquad -(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_m)(\Phi_m, (\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)})\Phi_m)\} \\ &+\sum_{(ij)}\{(\Phi_m, M_{ij}^{(2)+}(\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)}+\mathscr{M}_{ij}^{(2)})M_{ij}^{(2)}\Phi_n) \\ &\qquad +(\Phi_m, M_{ij}^{(2)+}(\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)}+\mathscr{M}_{ij}^{(2)})M_{ij}^{(2)}\Phi_m)(\Phi_m, \Phi_n) \\ &-(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_n)(\Phi_m, (\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)})\Phi_m) \\ &\qquad -(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_m)(\Phi_m, (\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)})\Phi_n)\}\end{aligned} \tag{29}$$

$$+\Big[(\Phi_m, \mathscr{M}^{(1)}\Phi_m)+\sum_{(ij)}\{(\Phi_m, M_{ij}^{(2)+}(\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)}+\mathscr{M}_{ij}^{(2)})M_{ij}^{(2)}\Phi_m)$$
$$-(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_m)(\Phi_m, (\mathscr{M}_i^{(1)}+\mathscr{M}_j^{(1)})\Phi_m)\}\Big]$$
$$\times\Big[(\Phi_m, \Phi_n)+\frac{2}{N}\sum_{(ij)}\{(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_n)$$
$$-(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_m)(\Phi_m, \Phi_n)\}+\ldots\Big]+\ldots.$$

Let us consider the special case

$$\delta_{mn}=\frac{(\Psi_m, \Psi_n)}{\sqrt{(\Psi_m, \Psi_m)(\Psi_n, \Psi_n)}} \tag{30}$$
$$=\frac{(\Phi_m, M^+M\Phi_n)}{\sqrt{(\Phi_m, M^+M\Phi_m)(\Phi_n, M^+M\Phi_n)}}=\sqrt{\frac{(\Phi_m, M^+M\Phi_n)}{(\Phi_m, M^+M\Phi_m)}\cdot\frac{(\Phi_m, M^+M\Phi_n)}{(\Phi_n, M^+M\Phi_n)}},$$

with $(\Phi_m, \Phi_m)=(\Phi_n, \Phi_n)=1$. We have

$$\frac{(\Phi_m, M^+M\Phi_n)}{(\Phi_m, M^+M\Phi_m)}=\left[\frac{\partial}{\partial\beta}\log(\Phi_m, M^+Me^{\beta\frac{\Phi_n}{\Phi_m}}\Phi_m)\right]_{\beta=0} \tag{31}$$
$$=(\Phi_m, \Phi_n)+\frac{2}{N}\sum_{(ij)}\{(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_n)-(\Phi_m, M_{ij}^{(2)+}M_{ij}^{(2)}\Phi_m)(\Phi_m, \Phi_n)\}+\ldots;$$

thus

$$\delta_{mn}=(\Phi_m, \Phi_n)+O\left(N\frac{\omega}{\Omega}\right). \tag{32}$$

As a first approximation, the Φ_m's can therefore be regarded as orthogonal.

Therefore, eq. (29) is an expansion series with respect to a quantity of the order $O(N\omega/\Omega)$: the true matrix element $(\Psi_m, \mathscr{M}^{(1)}\Psi_n)$ of a one-body operator $\mathscr{M}^{(1)}$ can be approximated by $(\Phi_m, \mathscr{M}^{(1)}\Phi_n)$ with an error of the order $O(N\omega/\Omega)$. On the other hand, the matrix element of a two-body operator $\mathscr{M}^{(2)}$ should be nearly $\sum_{ij}(\Phi_m, M_{ij}^{(2)+}\mathscr{M}_{ij}^{(2)}M_{ij}^{(2)}\Phi_n)$, which may considerably differ from $(\Phi_m, \mathscr{M}^{(2)}\Phi_n)$.

5. Discussion

The most important necessary condition for the validity of our method is (19). As seen from refs. [2]) and [5]), $N\omega/\Omega$ may have a magnitude of about $\frac{1}{8}$ in nuclear matter. Therefore, our method seems to be useful to understand the many apparently contraditory aspects of nuclear models. The most remarkable result is eq. (29) to calculate the matrix elements. The nearest-neighbour expansion [3]) seems to be essentially similar to our method and formally more rigorous, but our method might be sometimes more useful because of its fairly simple and intuitive character.

A plausible argument for the convergence of our pseudo-cluster expansion is

that the effective Hamiltonian H_0 involves only weak interactions and ensures the approximate freedom of particle motions in the asymptotic wave function Φ.

Though our method is also applicable to an imperfect Bose gas, we shall only mention nuclear problems here:

1) The weakness of the effective interactions in the effective Hamiltonian H_0 suggests both the appearance of the shell model aspects and the saturation property. The saturation condition should be investigated by the eq. (21) instead of the original eq. (3).

2) The foundation of the configuration mixing method [7]) is given by eqs. (21) and (29). It can easily be shown that our prescription involves short range correlation effects (higher cluster terms $O(N\omega/\Omega)$) as well as the configuration mixing effect due to the residual interactions in the asymptotic wave function Φ.

3) It is possible in principle to extend the Bethe-Goldstone equation to the case including the three-body correlations, to investigate the relation between the shell and collective models and the possibility of explaining the spin-orbit force in a nucleus by the three-body correlations or to examine the alpha-model, the superfluid model, the deuteron model, etc., from a unifying view point.

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THE ANGULAR DISTRIBUTION OF β PARTICLES

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Abstract: In this work a general formula is given for the angular distribution and polarization of electrons emitted in β decay from oriented nuclei. The formula is convenient for actual calculations on the one hand, and transparent enough to see in a straightforward way the influence of the various interaction types on the other hand. The result is also in a form convenient for switching it into the general formalism of angular correlations so as to get β—γ correlations (with or without polarization observation).

1. Introduction

Following the discovery of parity non-conservation in β decay much work has been done on the angular distribution and polarization of β particles [1]). In this work the angular distribution and polarization of β particles originating in transitions of any order of forbiddenness are treated in such a way that the similarity or difference of distributions resulting from different interaction combinations can be followed directly and easily.

The calculation is performed in several steps. Section 2 begins with some general remarks on decay processes and then the formula for the electron distribution is written down and the summation on neutrino states is performed. In sections 3 and 4 the formula for the angular distribution is worked out in more detail. The resulting expression (21) is factored into a few factors: a "nuclear factor" a "radial factor" and an "angular factor". The angular factor is independent both of the properties of the nuclei involved in the transition and of the interaction types (S, V etc). The nuclear factor depends only on whether the interaction is of the Fermi type (S, S′, V, V′, and also P, P′ and the relativistic parts of A and A′) or of the Gamow-Teller type (T, T′, A, A′ and the relativistic parts of V and V′). The detailed dependence on the interaction type appears only in the radial factor and has a very simple form, so that one can see immediately what modifications arise by choosing different interaction combinations. The radial factor is also the only factor depending on the electron energy.

In sections 5 and 6 summations on some magnetic quantum numbers appearing in eq. (21) are performed and the formula for the angular distribution is brought to the form (35), by which the main task of the work is finished. In section 7 the result (35) is rewritten in the form of a connection between the

statistical matrix describing the distribution of the nuclei among the initial states and the statistical matrix describing the distribution among the final states after a β particle has been emitted into a given direction (and possibly also into a given spin state). The result, eq. (45), is then specialized to the case of β—γ angular correlation when the β radiation comes from unoriented nuclei (eq. (46)), and to the case of angular distribution from oriented nuclei (eq. 47).

In section 8, the final results are summarized and discussed so as to enable a reader to perform calculations of angular distributions and polarizations without reference to the derivations given in the preceding sections.

2. General Discussion of Angular Distribution

The result of a measurement of a property of a decay product will depend linearly on time (just like the total decay probability) provided the matrix elements of the operator corresponding to this property have a δ-like singularity on the energy shell. In practice all properties of interest like spin and angular distribution, are of this type.

Let A be the observable to be measured, which has matrix elements of the form $\langle k_1|A|k_2\rangle = A_{k_1k_2}\delta(E_{k_1}-E_{k_2})$, where $A_{k_1k_2}$ is a regular function of the energies and other variables on which the states $\psi(k_1)$ and $\psi(k_2)$ of the decay products depend. The usual time-dependent perturbation theory shows that the increase in the expectation value of A per unit time is

$$\langle \dot{A}\rangle = \frac{2\pi}{\hbar}\sum_{k_1k_2}\langle 0|V|k_1\rangle A_{k_1k_2}\langle k_2|V|0\rangle\rho_E, \tag{1}$$

$\psi(0)$ being the state before the decay and ρ_E the density of states.

If we are interested in the angular distribution of the decay products, i.e. the probability of finding a particle moving outwards within a cone $\delta\Omega$ along the direction Ω we have to choose for A the operator $A(\Omega)$, where

$$A(\Omega) = \sum_{\alpha}|\Omega, \text{ out } \alpha\rangle\langle\Omega, \text{ out } \alpha|,$$

The sum being performed on all outgoing states.

If we have states ψ_i normalized within a sphere of radius R_{Norm} which have asymptotic expansions

$$\psi_i \to (1/r\sqrt{4\pi R_{\text{Norm}}})(F_i(\Omega)e^{ip_ir}+G_i(\Omega)e^{-ip_ir}),$$

we get for the matrix elements of A (omitting a δ-function of the energy difference)

$$A_{ij} = F_i^{\dagger}(\Omega)F_j(\Omega). \tag{2}$$

The matrix elements of the β^- decay interaction have the form

$$\begin{aligned}\langle \mathrm{e}, \tilde{\nu}, \mathrm{P}|V|\mathrm{N}\rangle &= \sum_{\mathrm{I}} C_{\mathrm{I}} \int_x (\phi_{\mathrm{f}}^\dagger(x) \sum_n O^{(\mathrm{I})}(n)\tau_+(n)\phi_{\mathrm{i}}(x))(\psi^\dagger(x)O^{(\mathrm{I})}\varphi(x)) \\ &= \sum_{\mathrm{I}} C_{\mathrm{I}} \int_x N^{\mathrm{I}}(x)(\psi^\dagger(x)O^{\mathrm{I}}\varphi(x)),\end{aligned} \tag{3}$$

the summation being performed on all interaction types ($\mathrm{I} = \mathrm{S}, \mathrm{V}, \ldots, \mathrm{S}', \ldots$); ϕ_{i} and ϕ_{f} are initial and final nuclear wave functions, $\psi(x)$ and $\phi(x)$ are the electron and antineutrino states; $O^{(\mathrm{I})}$ is some product of Dirac matrices depending on the interaction type.

The probability of finding within a unit time a β-particle moving outwards in the direction Ω is

$$\begin{aligned}W(\Omega) &= \langle \dot{A}(\Omega)\rangle \\ &= \frac{2\pi}{\hbar}\rho_E \sum_{\mathrm{II}'}\sum_{ij}\sum_{\tilde{\nu}} C_{\mathrm{I}} C_{\mathrm{I}'}^* \int_x\int_y N^{\mathrm{I}}(x)^*(\psi_i^\dagger(x)O^{(\mathrm{I})}\varphi_{\tilde{\nu}}(x))(\varphi_{\tilde{\nu}}^\dagger(y)O^{(\mathrm{I}')\dagger}\psi_j(y))N^{(\mathrm{I}')}(y)A_{ij}(\Omega),\end{aligned}$$

the summation being on all interaction types I, I', electron states i, j and antineutrino states $\tilde{\nu}$.

We choose for the $\tilde{\nu}$ states plane waves $\varphi_{\tilde{\nu}} = u(\mathbf{q}, \mathrm{s})^{i\mathbf{q}\cdot\mathbf{x}}$ and perform the summation on the two spin states by the usual trick: we introduce the projection operator $\Lambda_q = \frac{1}{2}(1+H_{\tilde{\nu}}/E) = \frac{1}{2}(1+\boldsymbol{\alpha}\cdot\mathbf{q}/q)$ before the function $\varphi_{\tilde{\nu}}(x)$ and sum on all four $\tilde{\nu}$-spin states. The summation on all $\tilde{\nu}$-states (if the $\tilde{\nu}$ direction is left unobserved) requires an integration on the direction of $\mathbf{q}$:

$$\begin{aligned}\frac{\hbar}{2\pi\rho_E} W(\Omega) = \sum_{\mathrm{II}'}\sum_{ij} C_{\mathrm{I}} C_{\mathrm{I}'}^* \int_x\int_y\int_q N^{(\mathrm{I})}(x)^* N^{(\mathrm{I}')}(y) \\ \cdot (\psi_i^\dagger(x)O^{(\mathrm{I})}\Lambda_q O^{(\mathrm{I}')\dagger}\psi_j(y))A_{ij}(\Omega)\exp i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y}).\end{aligned} \tag{4}$$

Expanding $\exp i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})$ in spherical harmonics the $\mathbf{q}$ integration can be performed. We get two types of terms: those originating from the first term in Λ_q (viz. $\frac{1}{2}$) will be called "terms of type A", and those originating from the second term of Λ_q (viz. $\frac{1}{2}\boldsymbol{\alpha}\cdot\mathbf{q}/q$) will be called "terms of type B".

Type A:

$$\tfrac{1}{2}\int_q \exp i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y}) = \tfrac{1}{2}(4\pi)^2 \sum_{\lambda\mu} j_\lambda(qx) j_\lambda(qy) \mathcal{Y}_{\lambda\mu}^*(x)\mathcal{Y}_{\lambda\mu}(y).$$

Type B:

$$\begin{aligned}&\tfrac{1}{2}\int (\boldsymbol{\alpha}\cdot\mathbf{q}/q)\exp i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y}) \\ &\quad = \gamma_5 \tfrac{1}{6}(4\pi)^2 \sum_{\lambda_1\lambda_2} i^{\lambda_1+\lambda_2}[(2\lambda_1+1)(2\lambda_2+1)]^{\frac{1}{2}}(\lambda_1 0;\, \lambda_2 0|10) j_{\lambda_1}(qx) j_{\lambda_2}(qy) \\ &\quad \times \sum_{\alpha\mu_1\mu_2} (-)^{\mu_2}(\lambda_1\mu_1;\, \lambda_2 -\mu_2|1\alpha).\end{aligned}$$

Using these expressions in eq. (4) we get

$$\frac{\hbar}{16\pi^3\rho_E}W(\Omega)$$

$$=\int_x\int_y\sum_{\mathrm{II}'}\sum_{ij}\sum_{\lambda\mu}C_\mathrm{I}C_{\mathrm{I}'}^*j_\lambda(qx)j_\lambda(qy)N^{(\mathrm{I})*}(x)N^{(\mathrm{I}')}(y)\left(\psi_i^\dagger(x)O^{(\mathrm{I})}O^{(\mathrm{I}')\dagger}\psi_j(y)\right)\mathscr{Y}_{\lambda\mu}^*(x)\mathscr{Y}_{\lambda\mu}(y)A_{ij}(\Omega)$$

$$+\tfrac{1}{3}i\sum_{\lambda_1\lambda_2}(-)^{\frac{1}{2}(\lambda_1+\lambda_2-1)}[(2\lambda_1+1)(2\lambda_2+1)]^{\frac{1}{2}}(\lambda_1 0;\,\lambda_2 0|10) \tag{5}$$

$$\times\int_x\int_y j_{\lambda_1}(qx)j_{\lambda_2}(qy)\sum_{\mathrm{II}'}\sum_{ij}\sum_{\mu_1\mu_2\alpha}C_\mathrm{I}C_{\mathrm{I}'}^*N^{(\mathrm{I})*}(x)N^{(\mathrm{I}')}(y)\left(\psi_i^\dagger(x)O^{(\mathrm{I})}\gamma_5\sigma_\alpha O^{(\mathrm{I}')\dagger}\psi_j(y)\right)\mathscr{Y}_{\lambda_1\mu_1}^*(x)\mathscr{Y}_{\lambda_2\mu_2}(y)$$

$$\times(-)^{\mu_2}(\lambda_1\mu_1;\,\lambda_2-\mu_2|1\alpha)A_{ij}(\Omega).$$

3. Electron Wave Functions and Angular Distribution Factors

It is well known that the solutions of the Dirac equation in a central field can be written in the form

$$\psi_{kjm}=\begin{pmatrix} i\,\chi^{ljm}\,f_k(r)\\ -\chi^{l'jm}g_k(r)\end{pmatrix}\quad\text{with}\quad k=\begin{cases}+(j+\frac{1}{2}) & l'=l+1=k\\ -(j+\frac{1}{2}) & l'=l-1=|k|;\end{cases} \tag{6}$$

χ^{ljm} is the spin-angular part of the usual two-component wave function of a Pauli-electron in a central field,

$$\chi^{ljm}=\sum_\alpha(lm-\alpha;\,\tfrac{1}{2}\alpha|jm)\mathscr{Y}_{l(m-\alpha)}\chi_\alpha;\quad \chi_{\frac{1}{2}}=\begin{pmatrix}1\\0\end{pmatrix},\quad \chi_{-\frac{1}{2}}=\begin{pmatrix}0\\1\end{pmatrix}.$$

The asymptotic behaviour of the radial functions f and g of an electron in a Coulomb field, normalized within a sphere of radius R_{Norm}, for small and big distances, is [2])

$$r\ll 1:\qquad \left.\begin{matrix}f_k\\ g_k\end{matrix}\right\}=a_j(Z,W)r^{\gamma-1}[(1\mp W)/W]^{\frac{1}{2}}\{e^{i\eta_k}(\gamma+iy)\mp\text{c.c.}\},$$

$$a_j(Z,W)=(1/R_{\mathrm{Norm}})^{\frac{1}{2}}2^{\gamma-1}e^{\frac{1}{2}\pi y}\frac{|\Gamma(\gamma+iy)|p^\gamma}{\Gamma(2\gamma+1)}, \tag{7}$$

$$r\gg 1:\qquad \left.\begin{matrix}f_k\\ g_k\end{matrix}\right\}=(1\mp W)^{\frac{1}{2}}\left(\frac{\pi}{W}\right)^{\frac{1}{2}}\frac{e^{i\delta_k}e^{ipr}}{r(4\pi R_{\mathrm{Norm}})^{\frac{1}{2}}}, \tag{8}$$

where

$$\gamma=[(j+\tfrac{1}{2})^2-(\alpha Z)^2]^{\frac{1}{2}},\quad y=\frac{\alpha Z}{v/c}=\frac{\alpha ZW}{p},\quad e^{2i\eta_k}=-\left(\frac{k-iy/W}{\gamma+iy}\right);$$

$$\delta_k=[y\ln 2pr-\arg\Gamma(\gamma+iy)-\tfrac{1}{2}\pi\gamma]+\eta_k.$$

For $\alpha Z\ll 1$ one gets, up to linear terms in αZ,

$$\eta_{k>0}=\tfrac{1}{2}\pi-\frac{(1+W)y}{2W(j+\frac{1}{2})},\quad \eta_{k<0}=\frac{(1-W)y}{2W(j+\frac{1}{2})}, \tag{9}$$

and hence one gets rather simple relations among the functions f, g in the region $r \ll 1$:

$$\begin{aligned} f_{(j+\frac{1}{2})} &= -a_j(0, W) r^{j-\frac{1}{2}}(2j+1)\left[\frac{W-1}{W}\right]^{\frac{1}{2}}, \\ g_{-(j+\frac{1}{2})} &= +a_j(0, W) r^{j-\frac{1}{2}}(2j+1)\left[\frac{W+1}{W}\right]^{\frac{1}{2}}, \\ f_{-(j+\frac{1}{2})} &= -a_j(0, W) r^{j-\frac{1}{2}} \alpha Z\left[\frac{W+1}{W}\right]^{\frac{1}{2}}, \\ g_{(j+\frac{1}{2})} &= -a_j(0, W) r^{j-\frac{1}{2}} \alpha Z\left[\frac{W-1}{W}\right]^{\frac{1}{2}}. \end{aligned} \tag{10}$$

If the longitudinal polarization of the electron is observed it is most convenient to use the following electron states:

$$\begin{aligned} \psi(W, j, m, \sigma_p = \pm 1) &= 2^{-\frac{1}{2}}\{e^{\frac{1}{2}i\eta_j}\psi_{-|k|jm} - \sigma_p e^{-\frac{1}{2}i\eta_j}\psi_{+|k|jm}\}, \\ \eta_j &= \delta_{(j+\frac{1}{2})} - \delta_{-(j+\frac{1}{2})}. \end{aligned} \tag{11}$$

The out-going waves in the asymptotic expressions of these wave functions for $r \gg 1$ are eigenfunctions of the operator $\boldsymbol{\sigma} \cdot \mathbf{p}/p$ with eigenvalues $\sigma_p = \pm 1$. It should be noted that $\boldsymbol{\sigma} \cdot \mathbf{p}/p$ commutes with the angular distribution operator $A(\Omega)$; hence the matrix elements (2) of $A(\Omega)$ are diagonal in σ_p. It is found to be convenient to use the following notation:

$$\begin{aligned} u_{(j+\frac{1}{2});j} &= e^{\frac{1}{2}i\eta_j} f_{-(j+\frac{1}{2})}, \quad u_{(j-\frac{1}{2});j} = e^{-\frac{1}{2}i\eta_j} f_{+(j+\frac{1}{2})}; \\ w_{(j-\frac{1}{2});j} &= e^{\frac{1}{2}i\eta_j} g_{-(j+\frac{1}{2})}; \quad w_{(j+\frac{1}{2});j} = e^{-\frac{1}{2}i\eta_j} g_{+(j+\frac{1}{2})}; \\ \eta_j &= \delta_{(j+\frac{1}{2})} - \delta_{-(j+\frac{1}{2})}, \qquad \zeta_j = \delta_{(j+\frac{1}{2})} + \delta_{-(j+\frac{1}{2})}; \end{aligned} \tag{12}$$

so that

$$\psi(jm\sigma_p) = 2^{-\frac{1}{2}} \begin{Bmatrix} i(\chi^{ljm} u_{l;j} - \sigma_p \chi^{(l-1)jm} u_{(l-1);j}) \\ -(\chi^{(l-1)jm} w_{(l-1);j} - \sigma_p \chi^{ljm} w_{l;j}) \end{Bmatrix} \quad (l = j + \tfrac{1}{2}), \tag{13}$$

and the outgoing part of the asymptotic expression of $\psi(jm\sigma_p)$ is

$$\psi(jm\sigma_p)_{\text{out}} = F_{jm\sigma_p} \frac{e^{ipr}}{r[4\pi R_{\text{Norm}}]^{\frac{1}{2}}} \tag{14}$$

where

$$F_{jm\sigma_p} = -2^{-\frac{1}{2}} e^{\frac{1}{2}i\zeta_j} \left(\frac{\pi}{W}\right)^{\frac{1}{2}} \begin{Bmatrix} (W-1)^{\frac{1}{2}}(\chi^{ljm} - \sigma_p \chi^{(l-1)jm}) \\ (W+1)^{\frac{1}{2}}(\chi^{(l-1)jm} - \sigma_p \chi^{ljm}) \end{Bmatrix}.$$

We can now calculate the matrix elements (2) of the angular distribution operator $A(\Omega)$. We shall refer to these as "angular factors". $A(\Omega)$ is as mentioned above diagonal in σ_p but is not diagonal in m and j. Non-diagonal terms

in m are of no interest if axial symmetry is preserved throughout the observation because these vanish on integration on the azimuthal angle around the axis of symmetry. We shall omit the indices σ_p and m in the designation of the matrix elements of $A(\Omega)$:

$$\begin{aligned} A(\Omega)_{j_1 j_2} &= (F^{\dagger}_{j_1 m_1 \sigma_p}, F_{j_2 m_2 \sigma_p}) = A^*(\Omega)_{j_2 j_1} \\ &= \pi e^{i(\zeta_{j_2}-\zeta_{j_1})} \sum_{ab} (-\sigma_p)^{j_1-j_2+a-b} (\chi^{aj_1 m_1 \dagger}, \chi^{bj_2 m_2}) \end{aligned} \tag{15}$$

writing out the product $(\chi^{aj_1 m_1 \dagger}, \chi^{bj_2 m_2})$, expanding the products of spherical harmonics

$$\mathscr{Y}_{a\alpha}(\Omega)\mathscr{Y}_{b\beta}(\Omega) = \sum_{k,q} \left[\frac{(2a+1)(2b+1)}{4\pi(2k+1)}\right]^{\frac{1}{2}} (a0, b0|k0)(a\alpha; b\beta|kq)\mathscr{Y}_{kq}(\Omega) \tag{16}$$

and performing the summation on the magnetic quantum numbers, one gets

$$A(\Omega)_{j_1 j_2} = \pi e^{i(\zeta_{j_2}-\zeta_{j_1})} \sum_{k,q} (-\sigma_p)^{j_1-j_2+k} \mathscr{Y}_{kq}(\Omega) \left[\frac{(2j_1+1)(2j_2+1)}{4\pi(2k+1)}\right]^{\frac{1}{2}} \tag{17}$$

$$\times (-)^{j_1+j_2+m_1-\frac{1}{2}} (j_1 - m_1; j_2 m_2|kq) \sum_{a,b} [(2a+1)(2b+1)]^{\frac{1}{2}} (a0; b0|k0) W(abj_1 j_2; k\tfrac{1}{2}).$$

4. First Rearrangement of Factors

To facilitate the explicit calculation of $W(\Omega)$ we shall rearrange eq. (5) and divide it into several factors which can be calculated separately.

The matrices $O^{(I)}O^{(I')\dagger}$ (appearing in terms of type A) and $O^{(I)}\gamma_5 O^{(I')\dagger}$ (appearing in type B-terms) can always be written as an outer product:

$$\begin{aligned} O^{(I)} O^{(I')\dagger} &= \rho^{(II')} \times \sigma^{(I)} \sigma^{(I')}, \\ O^{(I)} \gamma_5 O^{(I')\dagger} &= \rho^{(II')}_{\gamma_5} \times \sigma^{(I)} \sigma^{(I')}, \end{aligned}$$

where $\sigma^{(I)}$ is the unit matrix or one of the three Pauli matrices acting upon the upper two rows of $\psi(x)$ separately and upon the lower ones separately; $\rho^{(II')}$ is proportional to one of the four matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad -i\beta\gamma_5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

These matrices act on pairs of rows. We lump the matrix $\rho^{(II')}$ together with the radial functions iu and $-w$ into one factor $R^{(II')}(l_1 j_1, l_2 j_2)$, which is in fact the coefficient of $(-\sigma_p)^{(l_1-j_1)-(l_2-j_2)}(\chi^{l_1 j_1 m_1 \dagger}, \chi^{l_2 j_2 m_2})$ in $(\psi^{\dagger}(j_1 m_1 \sigma_p), \rho^{(II')}\psi(j_2 m_2 \sigma_p))$:

$$\begin{aligned} R^{(II')}(l_1 j_1, l_2 j_2) = \tfrac{1}{2}\{&(\rho^{(II')}_{11} u^*_{l_1 j_1} u_{l_2 j_2} + \rho^{(II')}_{22} w^*_{l_1 j_1} w_{l_2 j_2}) \\ &- \sigma_p (i\rho^{(II')}_{12} u^*_{l_1 j_1} w_{l_2 j_2} - i\rho^{(II')}_{21} w^*_{l_1 j_1} u_{l_2 j_2})\}. \end{aligned} \tag{18}$$

We shall refer to this last expression as the "radial factor". The radial factor of terms of type B, where an extra factor γ_5 is inserted into $\rho^{(II')}$, will be denoted by $R^{(II')}_{\gamma_5}(l_1 j_1, l_2 j_2)$.

Usually, in β-decay theory, one makes the approximation of neglecting the radial variation of the lepton wave functions within the nuclear volume. The functions $u_{lj}(r)$ and $w_{lj}(r)$ are replaced by the constants $u_{lj}(R_{\text{Nuc.}})$ and $w_{lj}(R_{\text{Nuc.}})$.

On the other hand, we lump together $\sigma^{(I)}\chi^{ljm}(x)$ with $\mathscr{Y}_{\lambda\mu}(x)$ and $N^{(I)}(x)$ into one factor. Here we have to distinguish between the case of a Fermi interaction (F), and the case of a Gamow-Teller interaction (G). In the F case, $\sigma^{(I)} = 1$ and we define a "nuclear factor" $\mathbf{K}_F$:

$$\mathbf{K}_F(ljm;\lambda\mu) = \int \phi_1^\dagger \Big(\sum_{\mathrm{n}} \chi^{ljm}(\mathrm{n})\mathscr{Y}_{\lambda\mu}(\mathrm{n})\tau_+(\mathrm{n})\Big)\phi_f \tag{19}$$

the summation on n being the summation on the nucleons. It should be noticed that $\mathbf{K}$ is, like χ^{ljm}, a two-rowed "vector". In the G-case $\sigma^{(I)}$ takes three values σ_x, σ_y, σ_z, as also does $N^{(I)}(x)$. We sum on these values and define a "nuclear factor" $\mathbf{K}_G$:

$$\mathbf{K}_G(ljm;\lambda\mu) = \sum_{k=1}^{k=3}\sigma_k \int \phi_1^\dagger \Big(\sum_{\mathrm{n}} \chi^{ljm}(\mathrm{n})\mathscr{Y}_{\lambda\mu}(\mathrm{n})\tau_+(\mathrm{n})\sigma_k(\mathrm{n})\Big)\phi_f. \tag{20}$$

In the case of a relativistic interaction ϕ_f is replaced by $\gamma_5\phi_f$ or $\beta\gamma_5\phi_f$, in the above definitions of the nuclear factors.

With the aid of the above definitions of the radial and nuclear factors, eq. (5) can be rewritten as follows:

$$\begin{aligned}\frac{\hbar}{16\pi^3\rho_E} W(\Omega) = {} & \sum C_I C_{I'}^* [(\mathbf{K}^\dagger_{(I)}(l_1 j_1 m_1;\lambda\mu)\cdot \mathbf{K}_{(I')}(l_2 j_2 m_2;\lambda\mu)] \\ & \times [j_\lambda{}^2(qR) R^{(II')}(l_1 j_1, l_2 j_2)][(-\sigma_p)^{(l_1-j_1)-(l_2-j_2)} A(\Omega)_{j_1 j_2}] \\ & + \tfrac{1}{3} i \sum C_I C_{I'}^* [(\mathbf{K}^\dagger_{(I)}(l_1 j_1 m_1;\lambda_1\mu_1)\sigma_\alpha \mathbf{K}_{(I')}(l_2 j_2 m_2;\lambda_2\mu_2)(\lambda_1\mu_1;\lambda_2 -\mu_2|1\alpha)](-)^{\frac{1}{2}(\lambda_1+\lambda_2-1)+\mu_2} \\ & \times [(2\lambda_1+1)(2\lambda_2+1)]^{\frac{1}{2}}(\lambda_1 0;\lambda_2 0|10)[j_{\lambda_1}(qR) j_{\lambda_2}(qR) R^{(II')}_{\gamma_5}(l_1 j_1, l_2 j_2)][(-\sigma_p)^{(l_1-j_1)-(l_2-j_2)} A(\Omega)_{j_1 j_2}].\end{aligned} \tag{21}$$

In this formula the following summations appear:

(I) on interaction types,

(II) on $l_1 l_2 \lambda_1 \lambda_2$ or λ (keeping $l+\lambda = n$ = order of forbiddenness for non relativistic interactions, and $l+\lambda = n-1$ for "relativistic interactions"),

(III) on j consistent with l,

(IV) on magnetic quantum numbers m_1, m_2, μ, μ_1, μ_2, α.

The advantage of formula (21) is that the nuclear factors are dependent only on the F or G character of the interactions (besides, they depend, obviously, on whether the interaction is "relativistic" or not). The more detailed dependence on the interaction type (e.g. whether it is S or V for a F-type interaction etc.) appears only in the radial factor $R^{(II')}(l_1 j_1, l_2 j_2)$. The whole energy depend-

ence is contained in the radial factors and the functions $j_\lambda(qR) = (qR)^\lambda/(2\lambda+1)!!$ which are concentrated in the second square bracket of (21). The angular dependence is contained in the last square bracket.

Our next step is to calculate the nuclear factors and to perform the summation on the magnetic quantum numbers.

5. The Nuclear Factors

As a preliminary step we calculate the components of $\mathbf{K}_F$ and $\mathbf{K}_G$. The definition (20) of $\mathbf{K}_F$ can be rewritten as follows:

$$(\mathbf{K}_F)_\alpha = \int \phi_1^\dagger (l\, m-\alpha; \tfrac{1}{2}\alpha|jm) \sum_{\mathrm{n}} \mathscr{Y}_{l,(m-\alpha)}(\mathrm{n}) \mathscr{Y}_{\lambda\mu}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_t .$$

Expanding the product of spherical harmonic (cf. (16)) we get

$$(K_F)_\alpha = \sum_{LM} (l\, m-\alpha; \tfrac{1}{2}\alpha|jm)(l\, m-\alpha; \lambda\mu|LM) \left[\frac{(2l+1)(2\lambda+1)}{4\pi(2L+1)}\right] (l0; \lambda 0|L0) \left(\int F_{LM}\right) \tag{22}$$

where

$$\left(\int F_{LM}\right) = \int \phi_1^\dagger \sum_{\mathrm{n}} \mathscr{Y}_{LM}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_t . \tag{23}$$

In a similar way we expand $(K_G)_\alpha$. Defining the nuclear matrix element

$$\left(\int G_{L'LM}\right) = \int \phi_1^\dagger \sum_{\mathrm{n}} \sum_{q} \sum_{\mu'} \sigma_q(\mathrm{n}) \mathscr{Y}_{L'M'}(\mathrm{n}) \tau_+(\mathrm{n}) (1q;\, L'M'|LM) \phi_t \tag{24}$$

and using the fact that the matrices $(\sigma_q)_{\alpha\beta}$ can be looked upon as the matrix of Clebsch-Gordan coefficients

$$(\sigma_q)_{\alpha\beta} = \sqrt{3}(\tfrac{1}{2}\beta;\, 1q|\tfrac{1}{2}\alpha) \tag{25}$$

we have

$$(K_G)_\alpha = \sum_{L'LM} \left(\int G_{L'LM}\right) \left[\frac{3(2l+1)(2\lambda+1)}{4\pi(2L'+1)}\right]^{\frac{1}{2}} (\lambda 0;\, l0|L'\,0)$$
$$\times \{ \sum_{q\beta M'} (-)^q (\tfrac{1}{2}\beta;\, 1-q|\tfrac{1}{2}\alpha)(1q; L'M'|LM)(\lambda\mu; l\,m-\beta|L'M')(l\,m-\beta;\tfrac{1}{2}\beta|jm)\}.$$

The summation in the curly bracket can be carried out and one gets

$$(K_G)_\alpha = \sum_{L',L,M} (-)^{m-\alpha+l+1} \left[\frac{3}{2\pi}(2l+1)(2\lambda+1)(2j+1)(2L+1)\right]^{\frac{1}{2}} (\lambda 0,\, l0|L'0)$$
$$\times \sum_{f,\varphi} X \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ l & L' & \lambda \\ j & L & f \end{pmatrix} (\tfrac{1}{2}-\alpha;\, \lambda\mu|f\varphi)(j-m;\, LM|f\varphi)\left(\int G_{L'LM}\right). \tag{26}$$

We proceed to calculate the combinations of nuclear factors appearing in (21).

TERMS OF TYPE A

F—F Interactions

$$\sum_{\mu}\left(\mathbf{K}_F^{\dagger}(l_1 j_1 m_1;\lambda\mu)\cdot\mathbf{K}_F(l_2 j_2 m_2;\lambda\mu)\right)$$

$$=\sum_{L_1 M_1}\sum_{L_2 M_2}\left(\int F_{L_1 M_1}\right)^{*}\left(\int F_{L_2 M_2}\right)\frac{2\lambda+1}{4\pi}\left[\frac{(2l_1+1)(2l_2+1)}{(2L_1+1)(2L_2+1)}\right]^{\frac{1}{2}}$$

$$(\lambda 0, l_1 0|L_1 0)(\lambda 0; l_2 0|L_2 0)\{\sum_{\mu}\sum_{\alpha}(\lambda\mu; l_1 m_1-\alpha|L_1 M_1)(l_1 m_1-\alpha;\tfrac{1}{2}\alpha|j_1 m_1)$$

$$\times(\lambda\mu; l_2 m_2-\alpha|L_2 M_2)(l_2 m_2-\alpha;\tfrac{1}{2}\alpha|j_2 m_2)\}\qquad(27)$$

$$=\sum\sum\left(\int F_{L_1 M_1}\right)^{*}\left(\int F_{L_2 M_2}\right)\frac{2\lambda+1}{4\pi}\left[\frac{(2l_1+1)(2l_2+1)}{(2L_1+1)(2L_2+1)}\right]^{\frac{1}{2}}(\lambda 0; l_1 0|L_1 0)(\lambda 0; l_2 0|L_2 0)$$

$$\times\sum_{f,\varphi}([[\lambda l_1]^{L_1}[l_2\tfrac{1}{2}]^{j_2}]^{j_1}|[[\lambda l_2]^{L_2}[l_1\tfrac{1}{2}]^{j_1}]^{f})(L_1 M_1; j_2 m_2|f\varphi)(L_2 M_2; j_1 m_1|f\varphi)$$

$$=\sum\sum\left(\int F_{L_1 M_1}\right)^{*}\left(\int F_{L_2 M_2}\right)\frac{2\lambda+1}{4\pi}[(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)]^{\frac{1}{2}}(\lambda 0; l_1 0|L_1 0)(\lambda 0; l_2 0|L_2 0)$$

$$\times\sum_{f\varphi}X\begin{pmatrix}\lambda & l_1 & L_1\\ l_2 & \frac{1}{2} & j_2\\ L_2 & j_1 & f\end{pmatrix}(L_1 M_1; j_2 m_2|f\varphi)(L_2 M_2; j_1 m_1|f\varphi).$$

G—G Interactions

$$\sum_{\mu}(\mathbf{K}_G^{\dagger}(l_1 j_1 m_1;\lambda\mu)\cdot\mathbf{K}_G(l_2 j_2 m_2;\lambda\mu))=\sum_{L_1' L_1 M_1}\sum_{L_2' L_2 M_2}\left(\int G_{L_1' L_1 M_1}\right)^{*}\left(\int G_{L_2' L_2 M_2}\right)$$

$$\times(-)^{m_1+m_2+l_1+l_2+1}3\frac{2\lambda+1}{2\pi}[(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)(2L_1+1)(2L_2+1)]^{\frac{1}{2}}\qquad(28)$$

$$\times(\lambda 0; l_1 0|L'_1 0)(\lambda 0; l_2 0|L'_2 0)\sum_{f\varphi}X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_1 & L'_1 & \lambda\\ j_1 & L_1 & f\end{pmatrix}X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda\\ j_2 & L_2 & f\end{pmatrix}(L_1-M_1; j_1 m_1|f\varphi)(L_2-M_2; j_2 m_2|f\varphi).$$

F—G Interactions

$$\sum_{\mu}\left(\mathbf{K}_F^{\dagger}(l_1 j_1 m_1;\lambda\mu)\cdot\mathbf{K}_G(l_2 j_2 m_2;\lambda\mu)\right)=\sum_{\mu}\left(K_G^{\dagger}(l_2 j_2 m_2;\lambda\mu)\cdot K_F(l_1 j_1 m_1;\lambda\mu)\right)^{*}$$

$$=\sum_{L_1 M_1}\sum_{L'_2 L_2 M_2}\left(\int F_{L_1 M_1}\right)^{*}\left(\int G_{L'_2 L_2 M_2}\right)(\tfrac{3}{2})^{\frac{1}{2}}\frac{2\lambda+1}{2\pi}(-)^{m_1+m_2+L_1+L'_2}\qquad(29)$$

$$\times[(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)(2L_2+1)]^{\frac{1}{2}}(\lambda 0; l_1 0|L_1 0)(\lambda 0; l_2 0|L'_2 0)$$

$$\times\sum_{f\varphi}X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda\\ j_2 & L_2 & f\end{pmatrix}W(f\tfrac{1}{2}L_1 l_1;\lambda j_1)(L_1-M_1; j_1 m_1|f\varphi)(L_2-M_2; j_2 m_2|f\varphi).$$

TERMS OF TYPE B

Using (25) the combination of nuclear factors appearing in the B-type terms of (21) can be rewritten

$$(\mathbf{K}_l \cdot \sigma \cdot \mathbf{K}_{l'}) \equiv \sum_{\alpha\beta} \sum_{\mu_1\mu_2} \sum_{q} K^*_{l\alpha} K_{l'\beta} (\sigma_q)_{\alpha\beta} (\lambda_1\mu_1; \lambda_2 - \mu_2 | 1q) (-)^{\mu_2}$$
$$= 3(-)^{\lambda_2} \sum\sum\sum K^*_{l\alpha} K_{l'\beta} (1q; \lambda_2\mu_2 | \lambda_1\mu_1) (\tfrac{1}{2}\beta; 1q | \tfrac{1}{2}\alpha) (2\lambda_1+1)^{-\frac{1}{2}}. \tag{30}$$

We proceed to perform the summations for the various interactions types F—F, G—G and F—G.

F—F Interactions

$$(\mathbf{K}_F \cdot \sigma \cdot \mathbf{K}_F) = \sum_{L_1 M_1} \sum_{L_2 M_2} \left(\int F_{L_1 M_1}\right)^* \left(\int F_{L_2 M_2}\right) \frac{3}{4\pi} (-)^{\lambda_2} \left[\frac{(2l_1+1)(2l_2+1)(2\lambda_2+1)}{(2L_1+1)(2L_2+1)}\right]^{\frac{1}{2}}$$
$$\times (l_1 0; \lambda_1 0 | L_1 0) (l_2 0; \lambda_2 0 | L_2 0) \{\sum\sum\sum (1q; \lambda_2\mu_2 | \lambda_1\mu_1) (\tfrac{1}{2}\beta; 1q | \tfrac{1}{2}\alpha)$$
$$\times (l_1 m_1 - \alpha; \tfrac{1}{2}\alpha | j_1 m_1) (l_2 m_2 - \beta; \tfrac{1}{2}\beta | j_2 m_2)$$
$$\cdot (l_1 m_1 - \alpha; \lambda_1\mu_1 | L_1 M_1) (l_2 m_2 - \beta; \lambda_2\mu_2 | L_2 M_2)\}.$$

The sum within the curly bracket can be completed into the recoupling coefficient [3])

$$([[(1\lambda_2)^{\lambda_1} l_1]^{L_1} (l_2 \tfrac{1}{2})^{j_2}]_\varphi^{f} | [(\lambda_2 l_2)^{L_2} [l_1 (\tfrac{1}{2} 1)^{\frac{1}{2}}]^{j_1}]_{\varphi'}^{f'})$$

by multiplying it by $(L_1 M_1; j_2 m_2 | f\varphi)(L_2 M_2; j_1 m_1 | f'\varphi')$.
Removing these extra factors again by the aid of the orthogonality relations of the Clebsch Gordan coefficients we get

$$(\mathbf{K}_F \cdot \sigma \cdot \mathbf{K}_F) = \sum_{L_1 M_1} \sum_{L_2 M_2} \left(\int F_{L_1 M_1}\right)^* \left(\int F_{L_2 M_2}\right) \frac{3}{4\pi} (-)^{\lambda_2} \left[\frac{(2l_1+1)(2l_2+1)(2\lambda_2+1)}{(2L_1+1)(2L_2+1)}\right]^{\frac{1}{2}}$$
$$\times (\lambda_1 0, l_1 0 | L_1 0) (\lambda_2 0; l_2 0 | L_2 0) \sum_{f\varphi} ([[(1\lambda_2)^{\lambda_1} l_1]^{L_1} (l_2 \tfrac{1}{2})^{j_2}]_\varphi^{f'} | [(\lambda_2 l_2)^{L_2} [l_1 (\tfrac{1}{2} 1)^{\frac{1}{2}}]^{j_1}]_\varphi^{f})$$
$$\times (L_1 M_1; j_2 m_2 | f\varphi) (L_2 M_2; j_1 m_1 | f\varphi). \tag{31}$$

The recoupling coefficient appearing here can be expressed in terms of "smaller" recoupling coefficients in various ways by performing the recoupling in steps. For example,

$$([[(1\lambda_2)^{\lambda_1} l_1]^{L_1} (l_2 \tfrac{1}{2})^{j_2}]_\varphi^{f} | [(\lambda_2 l_2)^{L_2} [l_1 (\tfrac{1}{2} 1)^{\frac{1}{2}}]^{j_1}]_\varphi^{f'})$$
$$= \sum_{x=L_2 \pm \frac{1}{2}} (-)^{1+\lambda_1+l_1+L_1} (2x+1) [2(2\lambda_1+1)(2j_1+1)(2j_2+1)(2L_1+1)(2L_2+1)]^{\frac{1}{2}}$$
$$\times W(l_1 \lambda_1 f j_2; L_1 x) W(l_1 j_1 x L_2; \tfrac{1}{2} f) X \begin{pmatrix} \lambda_2 & 1 & \lambda_1 \\ l_2 & \frac{1}{2} & j_2 \\ L_2 & \frac{1}{2} & x \end{pmatrix}. \tag{32}$$

G—G Interactions

$$(\mathbf{K}_G \cdot \sigma \cdot \mathbf{K}_G) = \sum_{L'_1 L_1 M_1} \sum_{L'_2 L_2 M_2} \left(\int G_{L'_1 L_1 M_1}\right)^* \left(\int G_{L'_2 L_2 M_2}\right) \frac{9\sqrt{2}}{2\pi} (-)^{l_1+l_2+\lambda_2+m_1+m_2}$$
$$\times [(2\lambda_1+1)(2\lambda_2+1)(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)(2L_1+1)(2L_2+1)]^{\frac{1}{2}}$$
$$\cdot (\lambda_1 0; l_1 0|L'_1 0)(\lambda_2 0; l_2 0|L'_2 0) \qquad (33)$$
$$\times \sum_{f\varphi} X \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ l_1 & L'_1 & \lambda_1 \\ j_1 & L_1 & f \end{pmatrix} X \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ l_2 & L'_2 & \lambda_2 \\ j_2 & L_2 & f \end{pmatrix} W(\tfrac{1}{2}1f\lambda_2; \tfrac{1}{2}\lambda_1)(L_1-M_1; j_1 m_1|f\varphi)(L_2-M_2; j_2 m_2|f\varphi).$$

F—G Interactions

$$(\mathbf{K}_F \cdot \sigma \cdot \mathbf{K}_G) = \sum_{L_1 M_1} \sum_{L'_2 L_2 M_2} \left(\int F_{L_1 M_1}\right)^* \left(\int G_{L'_2 L_2 M_2}\right) \frac{3\sqrt{3}}{2\pi} (-)^{m_1+m_2+l_2+\lambda_2+j_1-\frac{1}{2}}$$
$$\times [(2\lambda_1+1)(2\lambda_2+1)(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)(2L_2+1)]^{\frac{1}{2}}$$
$$\cdot (\lambda_1 0; l_1 0|L_1 0)(\lambda_2 0; l_2 0|L'_2 0) \qquad (34)$$
$$\times \sum_{f\varphi} X \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ l_2 & L'_2 & \lambda_2 \\ j_2 & L_2 & f \end{pmatrix} W(\tfrac{1}{2}\tfrac{1}{2}\lambda_1\lambda_2; 1f) W(\lambda_1 \tfrac{1}{2} L_1 j_1; fl_1)$$
$$\cdot (L_1-M_1; j_1 m_1|f\varphi)(L_2-M_2; j_2 m_2|f\varphi).$$

6. Second Rearrangement of Factors

In order to bring formula (21) to its final form we combine the nuclear expressions (27, 28, 29, 31, 33, 34) with the angular distribution factors (17) and perform the summation on m. The final result can be written as follows:

$$\frac{\hbar}{\pi^2 \rho_E} W(\Omega) = \sum C_1 C_{1'}^* (-\sigma_p)^{l_1+l_2+k} [e^{i(\zeta_{j_2}-\zeta_{j_1})} j_\lambda^2(qR) R^{(II')}(l_1 j_1, l_2 j_2)]$$
$$\times \left\{ \begin{array}{l} A_{FF}(\lambda; l_1 j_1 l_2 j_2; L_1 L_2 k) \sum_{M_1 M_2} \left(\int F_{L_1 M_1}\right)^* \left(\int F_{L_2 M_2}\right) \\ A_{GG}(\lambda; l_1 j_1 l_2 j_2; L'_1 L_1 L'_2 L_2 k) \sum_{M_1 M_2} \left(\int G_{L'_1 L_1 M_1}\right)^* \left(\int G_{L'_2 L_2 M_2}\right) \\ A_{FG}(\lambda, l_1 j_1 l_2 j_2; L_1 L'_2 L_2 k) \sum_{M_1 M_2} \left(\int F_{L_1 M_1}\right)^* \left(\int G_{L'_2 L_2 M_2}\right) \end{array} \right\}$$
$$\times (-)^{M_1} (L_1-M_1; L_2 M_2|kq) \left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}} \mathscr{Y}_{kq}(\Omega) +$$

$$+i\sum C_{\mathrm{I}}C^{*}_{\mathrm{I}'}(-\sigma_{\mathrm{p}})^{l_1+l_2+k}[e^{i(\zeta_{j_2}-\zeta_{j_1})}j_{\lambda_1}(qR)j_{\lambda_2}(qR)R^{(\mathrm{II}')}_{\gamma_s}(l_1j_1l_2j_2)]$$

$$\times\begin{Bmatrix} B_{\mathrm{FF}}(\lambda_1\lambda_2;\, l_1j_1l_2j_2;\, L_1L_2k)\sum\limits_{M_1M_2}\left(\int F_{L_1M_1}\right)^{*}\left(\int F_{L_2M_2}\right)\\ B_{\mathrm{GG}}(\lambda_1\lambda_2;\, l_1j_1l_2j_2;\, L'_1L_1L'_2L_2k)\sum\limits_{M_1M_2}\left(\int G_{L'_1L_1M_1}\right)^{*}\left(\int G_{L'_2L_2M_2}\right)\\ B_{\mathrm{FG}}(\lambda_1\lambda_2;\, l_1j_1l_2j_2;\, L_1L'_2L_2k)\sum\limits_{M_1M_2}\left(\int F_{L_1M_1}\right)^{*}\left(\int G_{L'_2L_2M_2}\right)\end{Bmatrix}$$

$$\times(-)^{M_1}(L_1-M_1;\, L_2M_2|kq)\left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}}\mathscr{Y}_{kq}(\Omega). \tag{35}$$

The summations here are on interaction types I and on l_1, l_2, λ_1, λ_2 (or λ) and j, like in (22), and also on $L_1L_2L'_1L'_2$ and k. The radial factors $R^{(\mathrm{II}')}(l_1j_1l_2j_2)$ are given by (18) and the coefficients $A_{\mathrm{II}'}$, $B_{\mathrm{II}'}$ are as follows:

$$\begin{aligned}&A_{\mathrm{FF}}(\lambda;\, l_1j_1l_2j_2;\, L_1L_2\,k)\\ &\quad=(2\lambda+1)(2j_1+1)(2j_2+1)[(2l_1+1)(2l_2+1)]^{\frac{1}{2}}(\lambda0;\, l_10|L_10)\\ &\quad\times(\lambda0;\, l_20|L_20)\sum_f(-)^{f-\frac{1}{2}}(2f+1)W(j_1j_2L_2L_1;kf)X\begin{pmatrix}\lambda & l_1 & L_1\\ l_2 & \frac{1}{2} & j_2\\ L_2 & j_1 & f\end{pmatrix}\sum_{ab}(j_1j_2k),\end{aligned} \tag{36}$$

$$\begin{aligned}&A_{\mathrm{GG}}(\lambda;\, l_1j_1l_2j_2;\, L'_1L_1L'_2L_2k)\\ &\quad=6(2\lambda+1)(2j_1+1)(2j_2+1)[(2l_1+1)(2l_2+1)(2L_1+1)(2L_2+1)]^{\frac{1}{2}}\\ &\quad\times(\lambda0;\, l_10|L'_10)(\lambda0;\, l_20|L'_20)(-)^{l_1+l_2+L_1+L_2+k}\\ &\quad\times\sum_f(-)^{f-\frac{1}{2}}(2f+1)W(j_1j_2L_1L_2;\,kf)X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_1 & L'_1 & \lambda\\ j_1 & L_1 & f\end{pmatrix}X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda\\ j_2 & L_2 & f\end{pmatrix}\sum_{ab}(j_1j_2k),\end{aligned} \tag{37}$$

$$\begin{aligned}&A_{\mathrm{FG}}(\lambda;\, l_1j_1l_2j_2;\, L_1L'_2L_2k)\\ &\quad=\sqrt{6}(2\lambda+1)(2j_1+1)(2j_2+1)[(2l_1+1)(2l_2+1)(2L_2+1)]^{\frac{1}{2}}\\ &\quad\times(\lambda0;\, l_10|L_10)(\lambda0;\, l_20|L'_20)(-)^{L'_2+L_2+k+1}\\ &\quad\times\sum_f(-)^{f-\frac{1}{2}}(2f+1)W(j_1j_2L_1L_2;kf)W(f\tfrac{1}{2}L_1l_1;\lambda j_1)X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda\\ j_2 & L_2 & f\end{pmatrix}\sum_{ab}(j_1j_2k);\end{aligned} \tag{38}$$

$$\begin{aligned}&B_{\mathrm{FF}}(\lambda_1\lambda_2;\, l_1j_1l_2j_2;\, L_1L_2k)\\ &\quad=[(2\lambda_1+1)(2\lambda_2+1)^2(2j_1+1)(2j_2+1)(2l_1+1)(2l_2+1)/(2L_1+1)(2L_2+1)]^{\frac{1}{2}}\\ &\quad\times(\lambda_10;\, l_10|L_10)(\lambda_20;\, l_20|L_20)(\lambda_10;\, \lambda_20|10)(-)^{\frac{1}{2}(\lambda_1-\lambda_2-1)}\sum_{ab}(j_1j_2k)\\ &\quad\times\sum_f([[(1\lambda_2)^{\lambda_1}l_1]^{L_1}(l_2\tfrac{1}{2})^{j_2}]^f|[(\lambda_2l_2)^{L_2}[l_1(\tfrac{1}{2}1)^{\frac{1}{2}}]^{j_1}]^f)(-)^{f-\frac{1}{2}}(2f+1)W(j_1j_2L_2L_1;\,kf)\end{aligned} \tag{39}$$

(the recoupling coefficient appearing here is given in terms of 6-j and 9-j symbols by (32)),

$$
\begin{aligned}
&B_{GG}(\lambda_1\lambda_2; l_1j_1l_2j_2; L'_1L_1L'_2L_2k)\\
&= 6\sqrt{2}(2\lambda_1+1)(2\lambda_2+1)(2j_1+1)(2j_2+1)[(2l_1+1)(2l_2+1)(2L_1+1)(2L_2+1)]^{\frac{1}{2}}\\
&\times (\lambda_1 0; l_1 0|L'_1 0)(\lambda_2 0; l_2 0|L'_2 0)(\lambda_1 0, \lambda_2 0|10)(-)^{\frac{1}{2}(\lambda_1+\lambda_2+1)+l_1+l_2+L_1+L_2+k}\sum_{ab}(j_1j_2k)\\
&\times \sum_f (2f+1)W(j_1j_2L_1L_2; kf)W(\tfrac{1}{2}\tfrac{1}{2}\lambda_1\lambda_2; 1f)X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_1 & L'_1 & \lambda_1\\ j_1 & L_1 & f\end{pmatrix}X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda_2\\ j_2 & L_2 & f\end{pmatrix}.
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
&B_{FG}(\lambda_1\lambda_2; l_1j_1l_2j_2; L_1L'_2L_2k) = 2\sqrt{3}(2\lambda_1+1)(2\lambda_2+1)(2j_1+1)(2j_2+1)\\
&\cdot[(2l_1+1)(2l_2+1)(2L_2+1)]^{\frac{1}{2}}(-)^{\frac{1}{2}(\lambda_1+\lambda_2-1)+L_1+L_2+l_1+l_2+k}\\
&\times(\lambda_1 0; l_1 0|L_1 0)(\lambda_2 0; l_2 0|L'_2 0)(\lambda_1 0; \lambda_2 0|10)\sum_{ab}(j_1j_2k)\\
&\times\sum_f(2f+1)W(j_1j_2L_1L_2; kf)W(\tfrac{1}{2}\tfrac{1}{2}\lambda_1\lambda_2; 1f)W(f\tfrac{1}{2}L_1l_1; \lambda_1j_1)X\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ l_2 & L'_2 & \lambda_2\\ j_2 & L_2 & f\end{pmatrix};
\end{aligned}
\tag{41}
$$

here we use the abbreviation $\sum_{ab}(j_1j_2k)$ for

$$\sum_{ab}[(2a+1)(2b+1)]^{\frac{1}{2}}(a0; b0|k0)W(abj_1j_2; k\tfrac{1}{2}).$$

7. Specialization of the Result (35) to the Cases of β—γ Angular Correlation and Distribution of β-Particles from Oriented Nuclei

The result (35) is in a form just suitable for the calculation of β—γ correlation when the first radiation (β) is emitted by oriented nuclei. The distribution of the nuclei among the initial states is represented by a statistical matrix $S_i(M'_i; M''_i)$. Using (35) we get the statistical matrix $S_f(M'_f; M''_f; \Omega)$ which describes the distribution of the nuclei among the final states provided a β-particle was emitted into the direction Ω (and spin state σ_p):

$$
\begin{aligned}
&S_f(M'_fM''_f; \Omega) = \sum_{II'}\sum_{L_1L_2k}U^{(II')}(L'_1L_1, L'_2L_2; k)\left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}}\mathcal{Y}_{kq}(\Omega)\\
&\times\Big\{\sum_{M_1M_2}\sum_{M'_iM''_i}(-)^{M_1}(L_1-M_1; L_2M_2|kq)\\
&\cdot\Big(\int\phi_{I_iM'_i}T_{L'_1L_1M_1}\phi_{I_fM'_f}\Big)^*\Big(\int\phi_{I_iM''_i}T_{L'_2L_2M_2}\phi_{I_fM''_f}\Big)S_i(M'_1M''_1)\Big\}
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&= \sum_{II'}\sum_{L'_1L_1L'_2L_2}{}' U^{(II')}(L'_1L_1, L'_2L_2; k)(I_i||T_{L'_1L_1}||I_f)^*(I_i||T_{L'_2L_2}||I_f)\left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}}\mathcal{Y}_{kq}(\Omega)\\
&\times\Big\{\sum_{M_1M_2}\sum_{M'_iM''_i}(-)^{M_1}S_i(M'_iM''_i)(L_1-M_1; L_2M_2|kq)(I_fM'_f; L_1M_1|I_iM'_i)(I_fM''_f; L_2M_2|I_iM''_i)\Big\}.
\end{aligned}
$$

Here, $\int T_{L'LM}$ denotes $\int F_{LM}$ or $\int G_{L'LM}$ according as (I) is a F or G interaction; $U^{(\mathrm{II}')}(L'_1 L_1 L'_2 L_2; k)$ is the coefficient of

$$\left(\int T_{L'_1 L_1 M_1}\right)^* \left(\int T_{L'_2 L_2 M_2}\right) (-)^{M_1} (L_1 - M_1;\ L_2 M_2 | kq) \left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}} \mathscr{Y}_{kq}(\Omega)$$

in (35), viz.

$$U^{(\mathrm{II}')}(L'_1 L_1, L'_2 L_2; k)$$
$$= \sum_{lj\lambda} C_l C^*_{l'} (-\sigma_p)^{l_1+l_2+k} e^{i(\zeta_{j_2}-\zeta_{j_1})} \{ j_\lambda{}^2(qR) R^{(\mathrm{II}')} A_{\mathrm{II}'} - i j_{\lambda_1}(qR) j_{\lambda_2}(qR) \times R^{(\mathrm{II}')}_{\gamma_5} B_{\mathrm{II}'} \}. \quad (43)$$

The relation (42) between the initial and final statistical matrices can be expressed most elegantly in terms of the statistical tensors defined as follows [4]):

$$S(t, \tau) = \sum_{M' M''} (-)^{I+M'} (I - M'; I M'' | t\tau) S(M' M''). \quad (44)$$

The summation on the magnetic quantum numbers can be performed and one gets

$$S_f(t\tau; \Omega) = \sum_{\mathrm{II}'} \sum_{L_1} \sum_{L_2} \sum_{kq} \sum_{s\sigma} U^{(\mathrm{II}')}(L'_1 L_1, L'_2 L_2; k) (I_1 || T_{L'_1 L_1} || I_f)^* (I_1 || T_{L'_2 L_2} || I_f)$$
$$\times [(2s+1)(2k+1)]^{\frac{1}{2}} (-)^{L_2+k+q} X \begin{pmatrix} I_1 & I_1 & s \\ L_1 & L_2 & k \\ I_f & I_f & t \end{pmatrix} (s\sigma; k-q | t\tau) S_1(s\sigma) \left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}} \mathscr{Y}_{kq}(\Omega). \quad (45)$$

In the usual case of β—γ angular correlation the initial distribution of the nuclei is random. In that case the initial statistical matrix has the form

$$S_i(M'_i M''_i) = \frac{\delta_{M'_i M''_i}}{2I_1+1} = \frac{(-)^{I_1+M'_1}}{(2I_1+1)^{\frac{1}{2}}} (I_i - M'_i;\ I_i M''_i | 00),$$

whence one sees immediately that all the statistical tensors vanish except the tensor of order zero: $S_1(s, \sigma) = (2I_1+1)^{\frac{1}{2}} \delta_{s,0} \delta_{\sigma,0}$. The summation on s, σ and k, q in (45) becomes trivial in that case and also the 9—j symbol reduces to a 6—j symbol. We get for the final statistical tensors

$$S_f(kq, \Omega) = \sum_{\mathrm{II}'} \sum_{L'_1 L_1} \sum_{L'_2 L_2} U^{(\mathrm{II}')}(L'_1 L_1 L'_2 L_2; k) (I_1 || T_{L'_1 L_1} || I_f)^* (I_1 || T_{L'_2 L_2} || I_f)$$
$$\times W(L_1 L_2 I_f I_f; k I_1) (-)^{L_1+L_2+I_1-I_f+q} (2I_1+1)^{-1} \left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}} \mathscr{Y}_{kq}(\Omega). \quad (46)$$

If we are interested in the angular distribution of β's emitted from oriented nuclei we have to sum (35) on the final states and to average on the initial states. The summation on the magnetic quantum numbers $M_1 M_2$, M_f can be performed at once. An alternative way to get the angular distribution is as follows: As the final states of the nuclei remain completely unobserved we

have to take the trace of the final statistical matrix. The trace of the matrix is just equal to $(2I_f+1)^{\frac{1}{2}}$ times the statistical tensor of order zero:

$$\frac{\hbar}{\pi^2\rho_E}W(\Omega) = (2I_f+1)^{\frac{1}{2}}S_f(00;\Omega)$$

$$= \sum_{\mathrm{II}'}\sum_{L'_1L_1}\sum_{L'_2L_2}\sum_{kq}U^{(\mathrm{II}')}(L'_1L_1L'_2L_2;k)(I_1||T_{L'_1L_1}||I_f)^*(I_1||T_{L'_2L_2}||I_f)\,W(L_1L_2I_1I_1;kI_f) \quad (47)$$

$$\times(-)^{I_f-I_1+k}S_1(kq)\left[\frac{4\pi}{2k+1}\right]^{\frac{1}{2}}\mathcal{Y}_{kq}(\Omega).$$

8. Summary and Discussion

In this paragraph our results will be summarized and discussed so as to enable the reader to perform numerical calculations without reference to the rather lengthy derivations given in the preceding parts.

Eq. (45) gives our final result in its most general form, viz. as a connection between an initial and a final density matrix. This equation, or rather, its specialization (46) to the case of an initial isotropic distribution of the nuclei among the magnetic substates is in a form convenient for calculations of angular correlations involving β radiations. For the calculation of angular distributions of β particles emitted from oriented nuclei, eq. (45) was specialized to the form (47). The statistical tensors defining the orientation of the nuclei are defined by eq. (44) (cf. ref. [4]). In all calculations the longitudinal polarization of the electrons can be determined at once. If it is desired to calculate the longitudinal polarization of β-particles emitted by unoriented nuclei, eq. (47) should be further specialized by omitting all terms except the one with $k = q = 0$

Eqs. (45)—(47) contain some rather simple geometrical factors: recoupling coefficients, statistical tensors and a spherical harmonic depending on the direction of the β-radiation. The specific nuclear information on the transition is contained in the double barred matrix elements, which, for a transition from an initial state $\phi_1(I_1M_1;\pi_1)$ to a final state $\phi_f(I_fM_f;\pi_f)$, are defined as follows:

$$(I_1||T_{L'L}||I_f) = \delta_{L'L}\left[\frac{(2I_1+1)^{\frac{1}{2}}}{(I_fM_f;\,LM|I_1M_1)}\right]\int\phi_1^\dagger\sum_{\mathrm{n}}\mathcal{Y}_{LM}(\mathrm{n})\tau_+(\mathrm{n})\phi_f \quad (48)$$

for an F-transition, and

$$(I_1||T_{L'L}||I_f) = \left[\frac{(2I_1+1)^{\frac{1}{2}}}{(I_fM_f,\,LM|I_1M_1)}\right]$$

$$\times\int\phi_1^\dagger\sum_{\mathrm{n}}\sum_{q=-1}^{+1}\sum_{M'=-L'}^{+L'}\sigma_q(\mathrm{n})\mathcal{Y}_{L'M'}(\mathrm{n})\tau_+(\mathrm{n})\phi_f(1q;\,L'M'|LM)$$

for a G transition. In the case of a relativistic transition a factor γ_5 or $\beta\gamma_5$ appears before ϕ_f.

The essential information on the specific β-transition is contained in the factors $U^{(II')}(L'_1L_1,L'_2L_2;k)$ defined by eq. (43). We proceed to discuss the calculation of these factors. The first step is to find which terms may appear in the summations on l, j and λ. The following vector relations must exist among the various angular momenta quantum numbers:

$$\begin{aligned}
&\mathbf{I}_f = \mathbf{I}_1+\mathbf{L},\\
&\mathbf{L} = \mathbf{L}' \text{ (F transition)}, \quad \mathbf{L} = \mathbf{L}'+1 \text{ (G transition)};\\
&\mathbf{L}' = \mathbf{l}+\boldsymbol{\lambda};\\
&\mathbf{j} = \mathbf{l}+\tfrac{1}{2}, \quad \mathbf{j}_\nu = \boldsymbol{\lambda}+\tfrac{1}{2}, \quad \mathbf{L} = \mathbf{j}+\mathbf{j}_\nu;\\
&\mathbf{k} = \mathbf{L}_1+\mathbf{L}_2.
\end{aligned} \tag{49}$$

Here, $\mathbf{l}$ and $\boldsymbol{\lambda}$ are orbital angular momenta connected to the electron and neutrino respectively, $\mathbf{j}$ and $\mathbf{j}_\nu$ are total angular momenta of these particles; $\mathbf{L}$ is the multipolarity of the transition, which, in the case of a G transition, is composed of an orbital part $\mathbf{L}'$ and a spin part.

Besides, the following parity relations must exist:

$$\begin{aligned}
(-)^{L'} &= \quad \pi_1\pi_f \text{ for non-relativistic transitions,}\\
(-)^{L'} &= -\pi_1\pi_f \text{ for relativistic transitions.}
\end{aligned} \tag{50}$$

The terms which may appear in a given transition ($I_1\pi_1 \to I_f\pi_f$) are not all of the same order of magnitude, and one has to pick out the dominant ones. The leptonic wave functions (cf. eqs. (54, 55)) contain the factors $(\not{p}R)^{j-\frac{1}{2}}(\alpha Z)^{l+\frac{1}{2}-j}$ and $(qR)^\lambda$ ($\not{p}$ and q are the momenta of the neutrino and the electron respectively, R is the radius of the nucleus and $\alpha = 1/137$). As $\not{p}R$ and qR are much smaller than unity, the dominant terms are those with smallest possible $\lambda+j$. It should be noted that relativistic nuclear matrix elements are smaller than non relativistic ones by a factor of order of magnitude $v_n/c \ll 1$ (v_n being the velocity of a nucleon in the nucleus). In table I the dominant terms for the various transitions are listed. At the end of the table the largest relativistic terms, which are not dominant, in the transitions $\Delta I = 0, 1$; $\pi_1\pi_f = -1$ are added.

The factor $U^{(II')}(L'_1L_1, L'_2L_2; k)$ is a product of several factors:

$$\begin{aligned}
U^{(II')}(L'_1L_1, L'_2L_2; k) = &\sum_{l_1l_2}\sum_{j_1j_2} C_1C_{1'}^*(-\sigma_p)^{l_1+l_2+k}e^{i(\zeta_{j_2}-\zeta_{j_1})}\\
&\times\{\sum_\lambda A_{II'}(\lambda; l_1j_1l_2j_2; L'_1L_1L'_2L_2; k)R^{(II')}(l_1j_1l_2j_2)j_\lambda^2(qR)\\
&-i\sum_{\lambda_1}\sum_{\lambda_2} B_{II'}(\lambda_1\lambda_2; l_1j_1l_2j_2; L'_1L_1L'_2L_2; k)R^{(II')}_{\gamma_5}(l_1j_1l_2j_2)j_{\lambda_1}(qR)j_{\lambda_2}(qR)\};
\end{aligned}$$

C_I is the coupling constant of the interaction type I (cf. eq. (52)), σ_p is the sign of the longitudinal polarization of the β-particles, ζ_j is a phase shift of the electronic wave function defined by eqs. (12) and (8) (cf. also eq. (54)), A and B are numerical factors defined by eqs. (36), (37), (38), (39), (40) (41): they depend only on the F or G character of the interaction type. The A and B

TABLE 1

Transition	Interaction type	Multi-polarity L	Multi-polarity L'	j	l	λ	Relative order of magnitude of matrix element
Allowed transitions							
$\pi_i\pi_f = 1,\ \Delta I = 0$	F	0	0	$\frac{1}{2}$	0	0	1
$\pi_i\pi_f = 1,\ \Delta I = 0,1$	G.T.	1	0				
First forbidden transitions							
$\pi_i\pi_f = -1,\ \Delta I = 0$	G.T.	0	1	$\frac{1}{2}$	1	0	αZ
	G.T.	1	1				
$\pi_i\pi_f = -1,\ \Delta I = 1$	G.T.	1	1				
	F.	1	1				
Unique first forbidden transitions							
$\pi_i\pi_f = -1,\ \Delta I = 2$	G.T.	2	1	$\frac{3}{2}$	1	0	(pR)
				$\frac{1}{2}$	0	1	(qR)
Second forbidden transitions							
$\pi_i\pi_f = 1,\ \Delta I = 2$	F.	2	2	$\frac{1}{2}$	1	1	$(\alpha Z)(qR)$
	G.T.	2	2	$\frac{3}{2}$	2	0	$(\alpha Z)(pR)$
$\pi_i\pi_f = 1,\ \Delta I = 2$	$G.T._{rel.}$	2	1	$\frac{1}{2}$	1	0	$(\alpha Z)v_n/c$
Unique second forbidden transitions							
$\pi_i\pi_f = 1,\ \Delta I = 3$	G.T.	3	2	$\frac{5}{2}$	2	0	$(pR)^2$
				$\frac{3}{2}$	1	1	$(pR)(qR)$
				$\frac{1}{2}$	0	2	$(qR)^2$
Third forbidden transitions							
$\pi_i\pi_f = -1,\ \Delta I = 3$	F.	3	3	$\frac{5}{2}$	3	0	$(\alpha Z)(pR)^2$
	G.T.	3	3	$\frac{3}{2}$	2	1	$(\alpha Z)(pR)(qR)$
				$\frac{1}{2}$	1	2	$(\alpha Z)(qR)^2$
$\pi_i\pi_f = -1,\ \Delta I = 3$	$G.T._{rel.}$	3	2	$\frac{3}{2}$	2	0	$(\alpha Z)(pR)v_n/c$
				$\frac{1}{2}$	1	1	$(\alpha Z)(qR)v_n/c$
First forbidden relativistic transitions							
$\pi_i\pi_f = -1,\ \Delta I = 0$	$F_{rel.}$	0	0	$\frac{1}{2}$	0	0	v_n/c
$\pi_i\pi_f = -1,\ \Delta I = 0,1$	$G.T._{rel.}$	1	0				

coefficients possess the following symmetry properties:

$$\begin{aligned} A_{\mathrm{II}'}(\lambda; l_1 j_1 l_2 j_2; L'_1 L_1 L'_2 L_2; k) &= A_{\mathrm{I}'\mathrm{I}}(\lambda; l_2 j_2 l_1 j_1; L'_2 L_2 L'_1 L_1; k), \\ B_{\mathrm{II}'}(\lambda_1 \lambda_2; l_1 j_1 l_2 j_2; L'_1 L_1 L'_2 L_2; k) &= -B_{\mathrm{I}'\mathrm{I}}(\lambda_2 \lambda_1; l_2 j_2 l_1 j_1; L'_2 L_2 L'_1 L_1; k). \end{aligned} \tag{51}$$

The only factors of $U^{(\mathrm{II}')}(L'_1 L_1 L'_2 L_2; k)$ which depend on the specific interaction type are the "radial factors" $R^{(\mathrm{II}')}(l_1 j_1 l_2 j_2)$ and $R^{(\mathrm{II}')}_{\gamma_5}(l_1 j_1 l_2 j_2)$. The former is given by

$$\begin{aligned} R^{(\mathrm{II}')}(l_1 j_1 l_2 j_2) = {} & \tfrac{1}{2}(\rho^{(\mathrm{II}')}_{11} u^*_{l_1 j_1} u_{l_2 j_2} + \rho^{(\mathrm{II}')}_{22} w^*_{l_1 j_1} w_{l_2 j_2}) \\ & - \tfrac{1}{2} i \sigma_{\mathrm{p}} (\rho^{(\mathrm{II}')}_{12} u^*_{l_1 j_1} w_{l_2 j_2} - \rho^{(\mathrm{II}')}_{21} w^*_{l_1 j_1} u_{l_2 j_2}). \end{aligned}$$

In this expression $\rho^{(\mathrm{II}')}_{ij}$ are the elements of a matrix $\rho^{(\mathrm{II}')}$ which is the "σ-independent part" of the product $O^{(\mathrm{I})}(O^{(\mathrm{I}')})^\dagger$. One observes immediately that the matrix ρ must have one of the four forms

$$\rho^{(1)} = \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \rho^{(2)} = -i\beta\gamma_5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

$$\rho^{(3)} = \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \rho^{(0)} = 1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

so that for each combination $l_1 j_1 l_2 j_2$ there are in all four different radial factors $R^{(1)}$, $R^{(2)}$, $R^{(3)}$ and $R^{(0)}$. In table 2 the matrices ρ corresponding to the various combinations of interactions are listed. The table is based on the following form of the β-interaction:

$$\begin{aligned} H_\beta = {} & \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \beta(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{S}}(\psi^\dagger \beta \varphi) && + C_{\mathrm{S}'}(\psi^\dagger \beta \gamma_5 \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{V}}(\psi^\dagger \varphi) && + C_{\mathrm{V}'}(\psi^\dagger \gamma_5 \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \beta(\mathrm{n}) \boldsymbol{\sigma}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{T}}(\psi^\dagger \beta \boldsymbol{\sigma} \varphi) && + C_{\mathrm{T}'}(\psi^\dagger \beta \boldsymbol{\sigma} \gamma_5 \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \boldsymbol{\sigma}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{A}}(\psi^\dagger \boldsymbol{\sigma} \varphi) && + C_{\mathrm{A}'}(\psi^\dagger \boldsymbol{\sigma} \gamma_5 \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \beta(\mathrm{n}) \gamma_5(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{P}}(\psi^\dagger \beta \gamma_5 \varphi) && + C_{\mathrm{P}'}(\psi^\dagger \beta \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \gamma_5(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{A}}(\psi^\dagger \gamma_5 \varphi) && + C_{\mathrm{A}'}(\psi^\dagger \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \beta(\mathrm{n}) \gamma_5(\mathrm{n}) \boldsymbol{\sigma}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{T}}(\psi^\dagger \beta \gamma_5 \boldsymbol{\sigma} \varphi) && + C_{\mathrm{T}'}(\psi^\dagger \beta \boldsymbol{\sigma} \varphi)\} \\ & + \int \phi_{\mathrm{f}}^\dagger \sum_{\mathrm{n}} \gamma_5(\mathrm{n}) \boldsymbol{\sigma}(\mathrm{n}) \tau_+(\mathrm{n}) \phi_{\mathrm{i}} && \{C_{\mathrm{V}}(\psi^\dagger \gamma_5 \boldsymbol{\sigma} \varphi) && + C_{\mathrm{V}'}(\psi^\dagger \boldsymbol{\sigma} \varphi)\}. \end{aligned} \tag{52}$$

The last three rows of the interaction are the relativistic parts of the A, T and V interactions. In the tables these parts are referred to by appending an index R to the symbols A, T and V.

The factor $R^{(II')}_{\gamma_5}(l_1 j_1 l_2 j_2)$ which appears in the second term of $U^{(II')}(L'_1 L_1 L'_2 L_2; k)$ differs from $R^{(II')}(l_1 j_1 l_2 j_2)$ only through replacing $\rho^{(II')}$ by $\rho^{(II')}_{\gamma_5}$ which is the "σ-independent part" of $O^{(I)}\gamma_5(O^{(I')})\dagger$. The matrices $\rho^{(II')}_{\gamma_5}$ are listed in table 3. It should be noted that the $R^{(\rho)}(l_1 j_1 l_2 j_2)$'s have the following symmetery property:

$$R^{(\rho)}(l_1 j_1 l_2 j_2) = \{R^{(\rho\dagger)}(l_2 j_2 l_1 j_1)\}^*. \tag{53}$$

The wave functions u_{lj}, w_{lj} appearing in $R^{(II')}(l_1 j_1 l_2 j_2)$ were defined by eqs. (7), (12). Usually the approximations for small distances and small charge (cf. eqs. (9), (10)) will suffice. Their values on the nucleus are

$$\begin{aligned}
u_{j-\frac{1}{2},j} &= -e^{-\frac{1}{2}i\eta_j} a_j(0, W) \left[\frac{W-1}{W}\right]^{\frac{1}{2}} (2j+1) R^{j-\frac{1}{2}}, \\
u_{j+\frac{1}{2},j} &= +e^{\frac{1}{2}i\eta_j}\ a_j(0, W) \left[\frac{W+1}{W}\right]^{\frac{1}{2}} (2j+1) R^{j-\frac{1}{2}}, \\
w_{j+\frac{1}{2},j} &= -e^{\frac{1}{2}i\eta_j}\ a_j(0, W) \left[\frac{W+1}{W}\right]^{\frac{1}{2}} \alpha Z R^{j-\frac{1}{2}}, \\
w_{j-\frac{1}{2},j} &= -e^{-\frac{1}{2}i\eta_j} a_j(0, W) \left[\frac{W-1}{W}\right]^{\frac{1}{2}} \alpha Z R^{j-\frac{1}{2}},
\end{aligned} \tag{54}$$

with

$$a_j(0, W) = \left(\frac{1}{R_{\text{Norm}}}\right)^{\frac{1}{2}} 2^{j-\frac{1}{2}}(j-\tfrac{1}{2})!\, \frac{p^{j+\frac{1}{2}}}{(2j+1)!}, \qquad e^{i\eta_j} = (e^{-i\eta_j})^* = i\left[\frac{1-i\alpha Z}{p(j+\frac{1}{2})}\right],$$

$$\zeta_{j_1} - \zeta_{j_2} = -\pi(j_1 - j_2) + \frac{\alpha Z}{p}\left[\frac{1}{j_1+\frac{1}{2}} - \frac{1}{j_2+\frac{1}{2}}\right].$$

The energy of the neutrino enters into the last factors of $U^{(II')}(L'_1 L_1 L'_2 L_2; k)$, viz. $j_\lambda(qR)$, which can be approximated by

$$j_\lambda(qR) = \frac{(qR)^\lambda}{(2\lambda+1)!!} \tag{55}$$

One should note that B-type terms (the second term of $U^{(II')}(L'_1 L_1 L'_2 L_2; k)$) vanish except when $|\lambda_1 - \lambda_2| = 1$. Consequently, they never contribute dominating terms except in unique forbidden transitions. It should also be noticed that $(\boldsymbol{\sigma} \cdot \mathbf{p})$ commutes with j and, therefore, in the calculation of polarization of β-particles from unoriented nuclei only terms diagonal in j contribute. In these polarization calculations the contributions of B-type terms to unique forbidden transitions vanish.

As an example of the usefulness of the present formulation let us consider the following question: Is pure scalar coupling distinguishable from pure vector

TABLE 2

Interaction products $\rho^{(II')}$.

non-rel.		rel.		non rel. / rel.	F	V	S	V′	S′
					G	A	T	A′	T′
					F	A'_R	P′	A_R	P
					G	V'_R	T'_R	V_R	T_R
F	G	F	G	$O^{(I)}$	1	β	γ_5	$\gamma_5\beta$	
V	A	A'_R	V'_R	1	1	β	γ_5	$-\beta\gamma_5$	
S	T	P′	T'_R	β	β	1	$\beta\gamma_5$	$-\gamma_5$	
V′	A′	A_R	V_R	γ_5	γ_5	$-\beta\gamma_5$	1	β	
S′	T′	P	T_R	$\beta\gamma_5$	$\beta\gamma_5$	$-\gamma_5$	β	1	

$\rho^{(II')}$ is the σ-independent part of $O^{(I)}(O^{(I')})$ †.

In the table all interactions having the same σ- independent part of $O^{(I)}$ are listed in the same row or column. An index R near an interaction symbol designs the relativistic part of that interaction.

TABLE 3

Interaction products $\rho^{(II')}_{\gamma_5}$.

non-rel.		rel.		non rel. / rel.	F	V	S	V′	S′
					G	A	T	A′	T′
					F	A'_R	P′	A_R	P
					G	V'_R	T'_R	V_R	T_R
F	G	F	G	$O^{(I)}$	1	β	γ_5	$\gamma_5\beta$	
V	A	A'_R	V'_R	1	γ_5	$-\beta\gamma_5$	1	β	
S	T	P′	T'_R	β	$\beta\gamma_5$	$-\gamma_5$	β	1	
V′	A′	A_R	V_R	γ_5	1	β	γ_5	$-\beta\gamma_5$	
S′	T′	P	T_R	$\beta\gamma_5$	β	1	$\beta\gamma_5$	$-\gamma_5$	

$\rho^{(II')}_{\gamma_5}$ is the σ-independent part of $O^{(I)}\gamma_5(O^{(I')})$†

In the table all interactions having the same σ-independent part of $O^{(I)}$ are listed in the same row or column. An index R near an interaction symbol designs the relativistic part of that interaction.

coupling by angular distribution experiments? As we have $\rho^{(SS)} = \rho^{(VV)} = 1$ the contributions of terms of type A are identical in both cases. In all transitions, except for unique forbidden ones only terms of type A give dominant contributions and consequently the two types of interactions cannot be distinguished by measuring these transitions. But $\rho^{(SS)}_{\gamma_5} = -\gamma_5 = -\rho^{(VV)}_{\gamma_5}$, which means that terms of type B give opposite contributions and the two interaction types can be distinguished by measuring unique-forbidden transitions to which B-type terms do contribute.

If we measure the intensity of the electron current only those terms of eqs. (45—47) contribute in which no factor σ_p appears. On the other hand, if one measures the polarization of the electrons, only terms which have a factor σ_p contribute. This means that the first or second term of $R^{(II')}(l_1 j_1 l_2 j_2)$ contributes to the intensity according to (l_1+l_2+k) being even or odd.

It should be remarked that by invoking the invariance under time reflections of the nuclear forces it can be shown that the product of reduced nuclear matrix elements $(I_1||T_{L'_1 L_1}||I_f)^*(I_1||T_{L'_2 L_2}||I_f)$ is real or imaginary according to the parity of $L'_1+L'_2$ being even or odd. Even parity goes with real, and odd parity goes with imaginary numbers.

Relativistic interactions have the same reality properties as the non-relativistic ones, i.e., an insertion of a γ_5 of $\beta\gamma_5$ into the nuclear matrix elements does not change their relative phases.

On passing from β^- to β^+ decay one has to change, besides the sign of the charge Z the nuclear matrix elements to their complex conjugates, and also the coupling constants C_I to $\pm C_I^*$. The plus or minus sign goes together with the plus or minus sign in the "commutation relations" of the interaction matrix $O^{(I)}$ with the charge conjugation matrix C, i.e. with the sign in $CO^{(I)}C = \pm(O^{(I)})^*$.

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MAGNITUDE OF THE PION RENORMALIZATION CONSTANTS

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Abstract: It has recently been pointed out that within the exact formulation of renormalization theory the pion mass renormalization constant δm^2 for pion-nucleon coupling is either infinite or indeterminate. This result is shown in the following to hold also in the case of pion-photon coupling. A detailed investigation reveals that for pion-nucleon coupling δm^2 is indeterminate only under very special circumstances.

In a recent paper [1]) the pion mass renormalization constant δm^2 was derived for a renormalizable theory (with pion-nucleon coupling) by using the exact formulation of renormalization theory developed by Källen [2]) and Lehmann [3]). It was pointed out that δm^2 was either infinite or indeterminate. We shall here investigate the bearing of this result on the problem of whether the rigorous formulation of pion renormalization theory leads to a finite and consistent scheme. Our investigations are based on the following axioms in the context of relativistic quantum field theory: (a) the Lagrangian describes a renormalizable field theory; (b) the fields are coupled by a *local*, causal interaction.

We begin by considering the case of a neutral pseudoscalar meson field $\phi(x)$ interacting with a nucleon field $\psi(x)$. The usual renormalized interaction Lagrangian is given by

$$L_I = -\tfrac{1}{2}igZ_1[\bar{\psi}, \gamma_5\psi]\phi+\tfrac{1}{2}Z_2\delta M[\bar{\psi}, \psi]+\tfrac{1}{2}\delta m^2 Z_3\phi^2-\tfrac{1}{4}\lambda Z_3{}^2\phi^4. \qquad (1)$$

The total Lagrangian with the above interaction term leads to a completely renormalizable field theory [4]); $\lambda = \lambda_1-\delta\lambda$ and λ_1 is a finite coupling constant to be determined by experiment; $\delta\lambda$ is the counter term required by perturbation theory in order to obtain finite S-matrix elements to all orders [1, 5]). The modified propagator $\Delta'(x)$ is defined by the vacuum expectation value [3, 6])

$$\langle[\phi(x), \phi(x')]\rangle_0 = i\Delta'(x-x'). \qquad (2)$$

By using (2), the equations of motion and the equal time commutation relations satisfied by the Heisenberg operator $\phi(x)$, we derive the following expression for δm^2 [1]):

$$\delta m^2 = -Z_3\left(\int_0^\infty (\kappa^2-m^2)\rho(\kappa^2)\mathrm{d}(\kappa^2)+3\lambda\langle\phi^2\rangle_0\right). \qquad (3)$$

The renormalization constants Z_3^{-1} and Z_2^{-1} are given by [3, 6]

$$Z_3^{-1} = \int_0^\infty \rho(\kappa^2)\mathrm{d}(\kappa^2), \quad Z_2^{-1} = \int_0^\infty \rho_1(\kappa^2)\mathrm{d}(\kappa^2), \tag{4}$$

and

$$0 \leqq Z_3 < 1, \quad 0 \leqq Z_2 < 1.$$

The vacuum expectation value $\langle\phi^2(x)\rangle_0$ can be obtained in the following way[1])

$$\langle\phi^2(x)\rangle_0 = \lim_{x\to x'} \langle 0|\phi(x)\phi(x')|0\rangle = \lim_{x\to x'} \Delta'^{(+)}(x-x'). \tag{5}$$

It is known that the modified propagators $\Delta'(x)$, $\Delta'^{(+)}(x)$ are at least as singular on the light cone as the free field propagators [6, 7]. For $x_0 > 0$, we have

$$\Delta'^{(+)}(x) = \frac{\mathrm{i}}{4\pi}\delta(x^2)Z_3^{-1} + \quad \text{less singular terms.} \tag{6}$$

We can now rewrite (3) with the aid of (5) and (6):

$$\delta m^2 = -\lim_{y\to 0}\left[Z_3\int_0^\infty \kappa^2\rho(\kappa^2)\mathrm{d}(\kappa^2) - m^2 + \frac{3\lambda}{4\pi}\delta(y^2) + \text{ less singular terms}\right], \tag{7}$$

where $y = x - x'$.

If only one stable particle of mass m is associated with $\phi(x)$, then $\rho(\kappa^2)$ is of the form [3])

$$\rho(\kappa^2) = \delta(\kappa^2 - m^2) + \sigma(\kappa^2), \tag{8}$$

where $\sigma(\kappa^2)$ vanishes for $0 \leqq \kappa^2 < (2m)^2$, and $\sigma(\kappa^2)$ contains no δ-functions.

Let us assume that all the renormalisation constants are finite, and proceed to examine the cases when this assumption is contradicted. In particular, we assume that $Z_3^{-1} < \infty$. Because $\rho(\kappa^2) \geqq 0$, we see that

$$\int_0^\infty \kappa^2\rho(\kappa^2)\mathrm{d}(\kappa^2) \geqq 0. \tag{9}$$

Thus we arrive at the conclusion that δm^2 is infinite for $\lambda > 0$, irrespective of whether or not (9) is bounded. If $\lambda < 0$ and (9) diverges, we may obtain a finite δm^2. The assumption that Z_3^{-1} is bounded corresponds to the existence of an upper bound in the integral

$$0 \leqq \int_{4m^2}^\infty \sigma(\kappa^2)\mathrm{d}(\kappa^2). \tag{10}$$

If (9) diverges, then the following integral has no upper bound:

$$0 \leqq \int_{4m^2}^\infty \kappa^2\sigma(\kappa^2)\mathrm{d}(\kappa^2). \tag{11}$$

As we have assumed that our $\sigma(\kappa^2)$ leads to an upper bound in (10), it is clear that (11) will diverge only under very special circumstances (e.g. if $\sigma(\kappa^2)$

behaves as $1/\kappa^3$ or $1/\kappa^4$). Thus we can only hope to obtain a finite δm^2 if $\lambda < 0$ and $\sigma(\kappa^2)$ leads to a divergent integral in (11) (and a convergent integral in (10), in order to maintain a finite Z_3^{-1}). However, it is obvious that even if (11) diverges, δm^2 will be indeterminate in the present form of local field theory.

Let us now consider the case of pion-photon coupling. For a gauge invariant theory the renormalized variables are [4])

$$\phi(x) = Z_2^{-\frac{1}{2}}\phi_u(x), \quad A_\mu(x) = Z_3^{-\frac{1}{2}}A_\mu{}^u(x), \quad e = Z_3^{\frac{1}{2}}e_0. \tag{12}$$

In terms of these quantities the renormalized interaction Lagrangian is

$$L_I = ieZ_2A_\mu(\phi\partial_\mu\phi^* - \partial_\mu\phi\phi^*) + e^2Z_2A_\mu{}^2\phi\phi^* + \delta m^2Z_2\phi\phi^* - \tfrac{1}{4}\lambda Z_2{}^2\phi^{*2}\phi^2; \tag{13}$$

Z_2 is less than unity but is no longer positive definite; Z_3 still satisfies $0 \leqq Z_3 < 1$. By means of a procedure similar to the preceding case [1]), we obtain the following expression for δm^2:

$$\delta m^2 = -Z_2\int_0^\infty \kappa^2\rho_1(\kappa^2)\mathrm{d}(\kappa^2) + e^2\langle A_\mu{}^2\rangle_0 - \lambda Z_2\langle\phi\phi^*\rangle_0 + \ldots, \tag{14}$$

where we have omitted additional finite terms. The expressions $\langle A_\mu{}^2\rangle_0$ and $\langle\phi\phi^*\rangle_0$ are again divergent due to the singularity on the light cone. $\langle\phi\phi^*\rangle_0$ and $\rho_1(\kappa^2)$ are no longer positive definite. As before, we see that δm^2 is either infinite or indeterminate.

Within the framework of the Heisenberg representation there is no obvious and satisfactory way of removing the singular expressions $\langle\phi^2\rangle_0$. It may be possible to remove the indicated inconsistency by introducing a non-local interaction in the Heisenberg representation. Alternatively, we could introduce an indefinite metric in Hilbert space and associate negative probabilities with some auxiliary fields which are coupled to the original system. However, both these possibilities possess well-known difficulties in their physical interpretation [8]).

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ON THE CONSERVED POLAR VECTOR CURRENT IN BETA-DECAY

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Abstract: It is suggested that if the polar vector current in beta-decay is not conserved, mesonic exchange effects may be appreciable in $0^+ \rightarrow 0^+$ transitions. Experimental data at present indicate that such effects are not present to more than 5 % in $O^{14} \rightarrow N^{14}$.

1. Introduction

There is now a large body of evidence [1]) that the decay of the μ-meson can be accounted for in terms of a V−A four fermion interaction of strength $g \approx 1.41 \times 10^{-49}$ erg · cm^3 which we denote symbolically by g(V−A). Similarly, nuclear beta-decay can be satisfactorily described by an effective interaction g'(V−λA) where $\lambda \approx 1.25$ and $g' \approx g$. This being so, it is an attractive idea to describe these decays in terms of a universal four fermion interaction between bare particles having the form g(V−A) and to ascribe the modification of the beta-decay interaction to the strong interactions experienced by a nucleon. However, although the A part is stronger for beta-decay, there seems to be no renormalization of the V part ($g \approx g'$) due to the strong interactions. This fact led Feynman and Gell-Mann [2]) to point out that renormalization of the V part can be removed (apart from small electromagnetic effects) if the four fermion interaction is regarded as due to a self-interacting current J_μ, the V part of which, $J_\mu^{\,V}$, is conserved. The situation then corresponds to the absence of electric charge renormalization due to strong interactions. Furthermore, there are then no mesonic exchange effects [3, 4]) for the V part in an allowed transition. This means that g is the same for the decay of a complex nucleus as for the neutron decay.

In the case of the axial vector interaction, although the fact that $\lambda \neq 1$ can be attributed to renormalizing effects of strong interactions, it is difficult to see why the effect is so small and takes the form of an enhancement [5]). In addition it is to be expected that there will be exchange effects for the A interaction in complex nuclei. These effects have been discussed in detail by Blin-Stoyle *et al.* [4]), who have shown that there is indeed some evidence for them.

The idea of a conserved vector current is an attractive one since it accounts

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for the fact that $g \approx g'$ in such a simple fashion. However, apart from this, there has so far been no definite experimental evidence put forward in support of the hypothesis, although a number of theoretical suggestions of possible experimental tests have been made [2, 6, 7]). The purpose of this paper is to point out another approach to the problem, namely, to consider the implication of a V interaction represented by a current which is *not* conserved. In this case there will be renormalization of the V part in beta decay and the agreement between g and g' would then seem to be fortuitous. On the other hand it is easy to account, at least in a qualitative fashion, for the value of λ [8]). There will also be exchange effects and it is primarily with these that this paper is concerned, the thesis being that in $0^+ \to 0^+$ transitions they are probably appreciable and that at present there is no indication of such effects. The conclusion might then be drawn that this indicates the conservation of J_μ^{V}.

2. Mesonic Exchange Effects in the V Interaction

In the paper by Blin-Stoyle *et al.* [4]) a fourth order perturbation calculation, using the pion-nucleon interaction theory with neglect of nucleon recoil, is carried out in order to estimate the magnitude and form of exchange effects in the case of the A interaction. A similar calculation has now been performed for an allowed transition in the case of the V interaction, assuming that the leptons are coupled only to the nucleon field. There is some ambiguity in this case, since, although in the case of the axial vector interaction we know how to treat the renormalization of the beta-decay vertex operator $\boldsymbol{\sigma\tau}$ (since it is identical with the nucleon-pion vertex) the same is not true for the polar vector vertex operator $\boldsymbol{\tau}$. We have therefore taken the experimentally observed (i.e. renormalized) strength for this operator. The calculation was then carried out exactly as in ref. [4]).

The exchange operator is found to have the form

$$(H_\beta^{\mathrm{V}})_{\mathrm{exch}} = g' \sum_{i \neq j} [f_{\mathrm{I}}(r) + f_{\mathrm{II}}(r)\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + f_{\mathrm{III}}(r) S_{ij}](\boldsymbol{\tau}_i^+ + \boldsymbol{\tau}_j^+) \tag{1}$$

where $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ are the spin and isobaric spin operators for the i^{th} nucleon,

$$S_{ij} = \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - \frac{3(\boldsymbol{\sigma}_i \cdot \mathbf{r})(\boldsymbol{\sigma}_j \cdot \mathbf{r})}{r^2},$$

and $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$.

In the approximation that there is no cut off in the momenta of virtual mesons other than in M_6 below [4]),

$$f_{\mathrm{I}} = -2f^4 \left[\frac{1}{x} + \frac{2}{x^2} + \frac{4}{x^3} + \frac{2}{x^4}\right] \mathrm{e}^{-2x}, \tag{2}$$

$$f_{\mathrm{II}} = \frac{2f^4}{3} \left[\left(\frac{4}{x^2} + \frac{4}{x^3} + \frac{2}{x^4}\right) \mathrm{e}^{-2x} - \frac{4}{3}\left(\frac{M_6}{\pi\mu}\right) \frac{\mathrm{e}^{-x}}{x}\right], \tag{3}$$

$$f_{III} = \frac{2f^4}{3}\left[\left(\frac{2}{x^2}+\frac{5}{x^3}+\frac{4}{x^4}\right)e^{-2x}+\frac{4}{3}\left(\frac{M_6}{\pi\mu}\right)\frac{e^{-x}}{x}\right], \quad (4)$$

with

$$M_6 = \int_0^{k_{max}} \frac{k^4\,dk}{(\mu^2+k^2)^2}$$

and $x = \mu r$, where μ is the pion mass.

As discussed in ref. [4]) these expressions are to be used in conjunction with nuclear wave functions which vanish when any two nucleons approach to within a distance r_c of the order $0.5/\mu$.

Evaluating the contribution of these exchange terms to the beta-decay of the triton as in ref. [4]) it is found that the square of the Fermi matrix element is *decreased* by $\approx 20\,\%$. In the case of heavier nuclei, using a short range approximation [4]), the exchange effects can be represented by an effective one-body operator

$$(H_\beta^{\,V})_{\text{one body}} = g'(\tfrac{1}{2}F_I - \tfrac{3}{2}F_{II})\rho(r)\tau^+ \quad (5)$$

where

$$F_i = \int_{r_c}^{\infty} f_i(r)\,d^3r$$

and $\rho(r)$ is the density of the nucleus at the position of the decaying nucleon. For a uniform density distribution $\rho = A/\frac{4}{3}\pi R^3$, with $R = r_0 A^{\frac{1}{3}}$ ($r_0 = 1.2\times 10^{-13}$ cm), a decrease in the square of the Fermi matrix element of 10—15 % is obtained.

For the triton decay a decrease in the square of the Fermi matrix element means that the agreement between $(ft)_{\text{neutron}}$ and $(ft)_{\text{triton}}$ brought about by exchange effects in the Gamow-Teller matrix element [4]) is now no longer obtained. Of course the estimate of $\approx 20\,\%$ decrease is crude and this disagreement may not be very significant. However, it may be regarded as some slight evidence in favour of a conserved V current.

Before considering the implications of the above result for the $0^+ \to 0^+$ transitions in heavier nuclei, it is necessary to discuss briefly other small effects which could affect the ft values of these decays.

3. Small Effects in $0^+ \to 0^+$ Transitions

The $0^+ \to 0^+$ transitions under consideration are listed in table 1 together with their ft-values and energies. In each of these transitions there are small contributions not normally considered in allowed transitions and we discuss each of these in turn.

3.1. SECOND FORBIDDEN CONTRIBUTIONS TO MATRIX ELEMENTS

The second forbidden contribution introduces terms involving the scalar matrix elements $\int r^2$ and $\int \boldsymbol{\alpha}\cdot\mathbf{r}$. The latter term vanishes for each of the three transitions if one makes the replacement

$$\int \boldsymbol{\alpha}\cdot\mathbf{r} \rightarrow -\frac{3i}{2M}\int (1+\tfrac{2}{3}i\mathbf{r}\cdot\mathbf{p})$$

suggested by Rose and Osborn [9]) and evaluates the matrix element using shell model wave functions.

TABLE 1

Decay	E (keV)	ft (sec)	Reference
O^{14}	1810± 8	3103±62	[a])
Al^{26}	3218±10	3080±57	[b])
Cl^{34}	4495±20	3165±77	[b])

[a]) J. B. Gerhart, Phys. Rev. **109** (1958) 897
[b]) O. C. Kistner, private communication.

The $\int r^2$ term is of the order $\frac{1}{3}k^2\int r^2$ where k is the lepton energy and for each decay this term causes a reduction in the transition probability. An upper limit to the magnitude of this contribution can be estimated by taking $k = W_0$ where W_0 is the end-point energy, and taking $\int r^2 = R^2$ where R is the nuclear radius. These upper limits are given in column 2 of table 2.

TABLE 2

Decay	Corrections (%)				Total (%)	Exchange Effect (%)		Grand Total (%)	
	3.1	3.2	3.3	3.4		(a)	(b)	(a)	(b)
O^{14}	0.02	0.01	0.1	−2±0.05	−1.9±0.05	5	10	3.1±0.05	8.1±0.05
Al^{26}	0.1	0.01	−1.0	−2±0.05	−2.9±0.05	3	6	0.1±0.05	3.1±0.05
Cl^{34}	0.2	0.01	−1.0	−2±0.05	−2.8±0.05	3	6	0.2±0.05	3.2±0.05

3.2. RELATIVISTIC CORRECTIONS TO MATRIX ELEMENTS

The beta-decay interaction operator is in relativistic form, involving Dirac operators and four component wave functions. In evaluating nuclear matrix elements the usual procedure is to go over to a non-relativistic form for the interaction by putting the Hamiltonian into "even" form by means of a canonical transformation which removes "odd' terms to any required order of $1/M$, where M is the nucleon mass [9]).

In the case of the $0^+ \rightarrow 0^+$ transitions, which are allowed and involve only the V part of the interaction, the Hamiltonian is

$$H = H_N + H_L + H_\beta$$

where

$$H_N = -\beta M - \boldsymbol{\alpha}\cdot\mathbf{p},$$

$$H_{\mathrm{L}} = \text{free lepton Hamiltonian},$$

$$H_{\beta} = g' L_4 \tau^+.$$

In the above expressions L_4 is the fourth component of the Lepton covariant $\bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_\nu$ and the Hamiltonian H_{N} has been taken as that for a free nucleon (the limitation of this assumption is discussed later).

The only odd nucleon operator is $-\boldsymbol{\alpha}\cdot\mathbf{p}$ and the Hamiltonian can be made even to order $1/M^2$ by the transformation

$$H' = \mathrm{e}^{iS_2}\mathrm{e}^{iS_1} H \mathrm{e}^{-iS_1}\mathrm{e}^{-iS_2}$$

with

$$iS_1 = \frac{-\beta}{2M}(-\boldsymbol{\alpha}\cdot\mathbf{p}),$$

$$iS_2 = \left(\frac{-\beta}{2M}\right)^2 [-\boldsymbol{\alpha}\cdot\mathbf{p}, H_\beta],$$

which results in the even Hamiltonian

$$H' = -\beta M - \frac{\beta}{2M}p^2 + H_\beta + \frac{1}{8M^2}[[\boldsymbol{\alpha}\cdot\mathbf{p}, H_\beta], \boldsymbol{\alpha}\cdot\mathbf{p}]. \tag{6}$$

The beta interaction thus has the correction term

$$\frac{1}{8M^2}[[\boldsymbol{\alpha}\cdot\mathbf{p}, L_4], \boldsymbol{\alpha}\cdot\mathbf{p}].$$

If it is assumed that L_4 is constant over the nuclear volume, this correction vanishes [3]). Without this assumption, the commutator becomes

$$\frac{1}{4M^2}[i\boldsymbol{\gamma}\cdot(\mathbf{k}\times\mathbf{p}) - \tfrac{1}{2}k^2] L_4 \tau^+$$

where $\mathbf{k}$ and $\mathbf{p}$ are the lepton and nucleon momenta respectively.

Inserting orders of magnitude for k and p shows that the contribution to the ft-value in the most unfavourable case is < 0.01 %. Of course, taking H_{N} to be the Hamiltonian for a free nucleon is very restrictive. However, if a nucleon potential U or βU had been included, U being a function of the nucleon position, the result would have been unchanged (since both terms are even), and only the inclusion of other odd terms would have a modifying effect. Even so, there would still be no relativistic correction so long as L_4 was assumed constant and the only effect would come from variation of the leptonic function over the nucleus. It would be surprising if these effects were appreciably larger than the figure (< 0.01 %) already obtained.

3.3. COULOMB EFFECTS

The matrix elements for an allowed pure Fermi transition are normally calculated on the assumption that nuclear forces are charge independent, and that isobaric spin is a good quantum number. This is a good approximation but small effects are nevertheless to be expected because of the influence of Coulomb forces. These effects have been considered in some detail by MacDonald [10]) for O^{14} and Cl^{34}. He finds that in the former case the *ft*-value is increased by ≈ 0.1 % and in the latter it is decreased by ≈ 1 %. One might guess that in Al^{26} (which has the same configuration of s and p nucleons as Cl^{34} and in which it is again a d nucleon which is decaying) the effect will also be of the order of magnitude 1 %. These figures are included in table 2.

3.4. ELECTROMAGNETIC RADIATIVE CORRECTIONS

These have been estimated by Berman [11]) and Kinoshita and Sirlin [12]) to be approximately the same for each nucleus and with a magnitude ≈ -2 % (in the *ft*-value). For the purposes of this paper, the main interest is not in the absolute magnitude of this correction (which we shall take to be nominally -2 % in O^{14}) but in its variation from one decay to another.

Kinoshita and Sirlin [12]) obtain for the percentage change in the *ft* value the expression

$$\frac{\delta(ft)}{ft} = -\frac{\alpha}{2\pi}\left[6\ln\left(\frac{\lambda}{M}\right) + 3\ln\left(\frac{M}{2W_0}\right) - 2.85\right] \times 100\ \% \qquad (7)$$

where λ is the ultra-violet cut-off, W_0 the maximum electron energy, M the proton mass and α the fine structure constant. The maximum variation of the effect from nucleus to nucleus will arise when $6 \ln (\lambda/M) = 2.85$. Even in this case the variation is less than 0.1 %. For our purposes, therefore, we shall take in table 2 the radiative corrections to be 2 ± 0.05 % for each decay.

4. Variation of Mesonic Exchange Effects

We now investigate the extent to which mesonic exchange effects differ in the three $0^+ \rightarrow 0^+$ transitions, using the approximate one body form of the exchange operator given in equation (5). The difference in the effect will result from the difference in the average $\langle \rho(r) \rangle$ over the position of the decaying nucleon, since in O^{14} a 1p nucleon decays, whereas in Al^{26} and Cl^{34} a 1d nucleon decays. An estimate of this effect has been made in the following way.

Oscillator functions $N_{1p} r \exp(-r^2/2a^2)$ and $N_{1d} r^2 \exp(-r^2/2a^2)$ are used for the 1p and 1d nucleon states, respectively, where the N's are appropriate normalizing factors and a is a measure of the extension of the nucleon orbit. The density distribution $\rho(r)$ is taken to have the form

$$\rho(r) = \rho_0 \exp(-r^2/\gamma a^2)$$

where γ determines the overall nuclear size. In O^{14}, for example, in order that the mean square radius of $\rho(r)$ should have the same value as that of its constituent particles (in 1s and 1p oscillator states), γ must take the value 1.4. We shall take γ to have this value for all three decays. A straightforward calculation then gives

$$\langle \rho(r) \rangle_{1l} = \left(\frac{\gamma}{\gamma+1}\right)^{\frac{2l+3}{2}} \frac{1}{r_0^3} \left(\frac{3}{2\pi}\right)^{\frac{3}{2}} \tag{8}$$

where the mean square radius of the density distribution has been put equal to $r_0 A^{\frac{1}{3}}$. Thus,

$$\frac{\langle \rho(r) \rangle_{1p}}{\langle \rho(r) \rangle_{1d}} = \frac{\gamma+1}{\gamma} = 1.7. \tag{9}$$

This means that if there is, for example, a 10 % exchange effect in O^{14}, then the effect in Al^{26} and Cl^{34} will be 6 %. In table 2 mesonic exchange corrections are included for two cases, namely, (a) an assumed 10 % correction and (b) an assumed 5 % correction in O^{14}. The total corrections for cases (a) and (b) including all the other small effects are given in the last two columns. It can be seen that with no exchange effect there is a maximum difference $\approx +(1\pm 0.1)\%$ between the *ft*-values of O^{14}, Al^{26} and Cl^{34}; with 5 % exchange effect in O^{14} the difference is $\approx +(3\pm 0.1)$ and with 10 % exchange effect in O^{14} the difference is $\approx +(5\pm 0.1)$ %.

Experimentally, taking Al^{26} and Cl^{34} together and comparing with O^{14} the difference in *ft*-values is $-(0.2^{+3.5}_{-3.2})$ %. Thus, the experimental *ft*-values are certainly consistent with there being no exchange effects. However, if the extreme experimental values are taken, they are not inconsistent with about 5 % exchange effect in O^{14}. This value is still less than the theoretical result estimated in section 2 and if any reliance could be placed on that result, we should have a clear indication that the V current is conserved. Unfortunately such a crude calculation can at most be taken to indicate the sign and order of magnitude of the effect. Clearly, however, the more accurately the *ft*-values of the three $0^+ \rightarrow 0^+$ transitions are shown to vary according to column 6 of table 2, the more certainly can it be said that there are no exchange effects and that the polar vector current is conserved.

One of the authors (R. J. B-S) is indebted to Professor H. Primakoff for a remark which led to this investigation. Another (J. S. T.) wishes to acknowledge the award of a Bryce Scholarship by Glasgow University.

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FLUCTUATIONS OF PARTIAL RADIATION WIDTHS OF U^{239}, Hf^{178} AND Pt^{196}

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Abstract: Partial radiation widths in U^{239}, Hf^{178} and Pt^{196} are shown to have significant fluctuations which are not always as large as those of the reduced neutron widths. One finds $J = 1$ for the 11.9- and 19.6-eV resonances of Pt.

For determining the spin J of a resonance neutron level the situation is favourable when a target nucleide in a $\frac{1}{2}$—state forms an even final nucleus. Then the ground state gamma-ray transition is forbidden when $J = 0$ and is of the strong E1 character when $J = 1$. The observation of capture gamma rays of energy equal to the neutron binding energy then indicates $J = 1$. The method has been employed for Hg^{200} and W^{184} product nucleides [1, 2]). The method requires (1) independent knowledge of the neutron binding energy, (2) sufficient energy resolution in the gamma-ray detector, sufficient knowledge of the low excited states (some of which may be missed in studies of radioactive decay), and sufficient separation of low levels of small spin to assure that a positive response of the high energy detector is caused by a ground state transition signifying $J \neq 0$ and (3) the assurance that a small high energy counting rate is not caused by a fluctuation in the strength of the favoured transition rather than caused by multipole order selection rules.

To obtain information about condition (3), high energy transitions in U^{239}, Hf^{178} and Pt^{196} were investigated. It has been assumed that the partial radiation widths have a distribution similar to that of neutron widths with their large fluctuations [3]). It would then be very difficult to infer spins from the high energy counting rates.

The neutron source was the 6-MeV electron linear accelerator with 1.2-μsec beam pulses at 112 sec^{-1} and an average beam current of 25 μA. The flight path was 3.3 m (because of intensity limitations) and the detector was a single 3-in. by 3-in diam cylindrical NaI(Tl) scintillator to minimize summing. The time channels were 2 μsec. A 2-cm neutron shield of paraffin and Li^6 was inserted between the sample and the scintillator. The pulse height from 4.42-MeV photons of RaD-Be was used for calibration and was stable within 1 per cent during a day.

† Supported by the U. S. Atomic Energy Commission.

The results for each resonance in table 1 are stated as N_H/N_L, i.e., the high channel counting rate divided by the low channel counting rate. The normalization for each isotope is arbitrary. N_L is approximately proportional to the number of neutrons captured in the resonance. To this extent N_H/N_L is proportional to the partial radiation width of the transitions contributing to N_H. More accurate estimation of the number of captured neutrons requires computation from resonance parameters, including corrections for multiple interactions in the thick samples, or correction of N_L for gross spectral variations (e.g., variations in multiplicity).

TABLE 1

Values of N_H/N_L (approximately proportional to the partial radiation width)

Compound nucleus	High channel (MeV)	Neutron energy (eV)	N_H/N_L
U^{239}	3.7—4.0	6.7	1.00 ± 0.11
U^{239}	3.7—4.0	21	0.77 ± 0.16
U^{239}	3.7—4.0	37	1.23 ± 0.22
Hf^{178}	6.6—7.3	1.1	1.00 ± 0.13
Hf^{178}	6.6—7.3	2.4	0.78 ± 0.10
Pt^{196}	7.1—7.8	11.9	1.00 ± 0.06
Pt^{196}	7.1—7.8	19.6	0.35 ± 0.06

The ratio N_H/N_L is arbitrarily normalized to unity for the resonance of lowest energy.

The equipment was checked with a 0.5-mm sample of tungsten. N_H/N_L was 1.00 ± 0.10, 0.47 ± 0.09 and 0.49 ± 0.07 for the 7.6-, 27- and 47-eV regions, respectively, in agreement with previous results [2]). The low channel was 2.7—3.4 MeV and the high channel was 6.7—7.4 MeV.

For uranium a 1-mm sample was used with a low channel of 1.5—1.8 MeV. The results of Campion *et al.* [4]) indicate that the high channel would be dominated by a 4.06-MeV transition of unknown multipole order which is not a ground state transition. Table 1 shows that this transition probability is very nearly the same for each of the three ($J = \frac{1}{2}$) resonances, unlike the reduced neutron widths.

For hafnium, a 0.25-mm sample was used with a low channel of 2.7—3.4 MeV. With the results of Campion and Bartholomew [5]) the estimated relative contributions to the high channel counting rate are 0.05 of 7.62 MeV, 0.04 of 7.33 MeV, 0.19 of 6.77 MeV, and < 0.05 of 6.44 MeV, so that the contribution of the 6.77 MeV transition dominates each of the others by a factor ≈ 4. Again the ratios N_H/N_L are quite similar. This is in contrast with the results [6]) with the low energy transitions in Hf^{178}. It is not known whether the J values of these two resonances are the same. The dependence of the counting rate on the position of the high channel was more compatible with the Hf results of

Campion and Bartholomew [5]) for photons above 6 MeV than with those of Groshev *et al.* [7]).

For platinum a 1-mm sample was used with a low channel of 2.7—3.4 MeV. For the energies and intensities from Groshev *et al.* [7]), the only contributions to the high channel counting rate are estimated as 0.3 units from transitions to the ground state, 0.1 units from transitions to the first 2+ state at 354 keV and 0.2 units from transitions to the second 2+ state at 686 keV. For a 1− resonance each of these transitions would be E1, while for 0− they would be forbidden and M2, respectively. Consequently, if the 11.9- and 19.6-eV resonances have $J = 1$ and $J = 0$, respectively, as concluded by Brussel and Fox [8]), then to account for the appreciable N_H/N_L at 19.6 eV there must be strong transitions to low energy levels presently unknown. However, these levels would probably be of spin 1 and the transitions should be seen in the thermal capture spectrum [7]). An alternative interpretation is that there are significant fluctuations in these partial radiation widths, as there are to a smaller degree in W^{184}.

In summary, for these examples, the partial radiation widths are not invariant, nor do they always have the extreme fluctuations of the reduced neutron widths [3]).

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BOOK REVIEWS

D. L. CONKLIN *et al.*, *Economics of Nuclear and Conventional Merchant Ships* (U. S. Government Printing Office, Washington 25, D.C.xi—869 p. $ 4.50)

This report was prepared by American Standard, Atomic Energy Division, at the request of U.S. Atomic Energy Commission, Maritime Reactors Branch (Contract No. AT (04-3)-109, Directive G). Undoubtedly, it represents the most extensive investigation of the subject published up till now. About 50 industrial concerns, shipowners, and institutions have taken part in the investigation.

Five principal types of ships have been examined (cargo liners, tankers, ore carriers, cargo-passenger liners and passenger liners) and within each type, the size and speed have been varied so that it has been possible to examine a total of 72 different ships, each with the following 5 different propelling machineries: conventional steam-turbine, pressurized-water reactor, boiling-water reactor, organic-moderated reactor and gas-cooled reactor. For each of these 360 combinations of ship and machinery, the costs of construction and operation have been calculated for 1957, 1965 and 1970, and 5 round trips of various lengths, various coefficients of utilization of the cargo capacity etc. have been considered. The final amount of combinations is about 30 000.

A detailed examination of the four reactor systems mentioned takes up almost one half of the report. Within each of them has been developed a "reactor family" which, from an economic aspect, formed the basis for selection of the optimum construction. These sections especially contain many details of great value for the estimation of how the price and weight of a reactor plant are composed, how an alteration of the parameters will influence the construction and operation costs, etc.

However, a considerable unreliability of the data, even those stated to be valid for the year 1957, should be taken into account. Let us give some examples. The insurance premium of nuclear ships are estimated at a rate 60 to 100 % higher than that of conventional ships: such a rate is bound to be a chance estimate owing to the fact that the insurance problems are completely unsettled. The shielding, which amounts to essentially more than 50 % of the weight of the reactor plant, presents another material unreliability. Apparently, no optimalization calculations have been made for this item, and for a specific plant the same shielding dimensions have been used, e.g., for tankers and passenger liners. Moreover, the directives stated for the PWR-plant do not seem to have been followed in the N/S Savannah project. The core geometry originally suggested for this vessel corresponds to geometry 'A', in the study of which the total costs per day are stated to be 10 % lower than for geometry 'B'. Yet, it is the latter which is being constructed. However, other aspects than economy may have caused this change.

The aim of these sporadic remarks is to emphasize that the conclusions stated in the book and so often quoted during the last few months seem rather hazardous. This applies particularly to the assumptions made for 1965 and 1970 for plants which are not at all developed to such a degree that they could be installed in a ship. For instance it has been stated that "The most attractive system for nuclear ships in the immediate future is the boiling-water reactor, and the most attractive system for the near future is the organic-moderated reactor". On closer examination, it appears that the figures which have led to this conclusion are those included in the tables I—XVIII. Now, one sees that, independently of the combination of the trade route distance, speed and ship considered, the differences in cost per ton of cargo by using reactor plants of the types PWR, BWR, and OMR generally turn out to be less than 1 % and never exceed 2 %. This and many other results of the investigation are in themselves so interesting that they should not be overshadowed by general and doubtful conclusions.

N. K.

J. McCONNELL, *Quantum Particle Dynamics* (North-Holland Publishing Company, Amsterdam, 1958. xi—252 p. 20 fl.)

The purpose of this work, as stated by the author in his preface, is "to provide an up-to-date book of convenient size that will enable those with no previous knowledge of relativity or quantum theory to acquire the fundamentals of quantum mechanics, quantum electrodynamics and meson theory, and apply them to the study of elementary particles".

This is indeed a programme of considerable scope. Since "no more than a working knowledge of calculus and of elementary algebra is presumed", most of the calculations presented are performed step by step in much detail. Considering the proportions of the subject-matter, one would thus expect a bulky work to be the result. Nevertheless, the first eight chapters, containing a survey of the main problems of relativity and quantum theory, take up a mere 168 pages; the remaining four chapters — Photoelectric Effects; Bremsstrahlung and Electron Pair Production; Nuclear Forces; Mesons and Nucleons — cover altogether 78 pages.

To press all these items into one modest-sized volume the author has had to make a strict selection of material. The reader is thus led along a very narrow path, the only deviations from which are quotations of results obtained by various research workers and here presented without due explanation of the way in which they were derived.

A question of principle here presents itself: Who are the people for whom this book was written, people with no knowledge of relativity or quantum theory and only a very rough idea of mathematical methods, and yet engaged in the study of elementary particles? To this question the author gives the following answer in the preface: "It is hoped that this book . . . will be found useful by cosmic ray research workers who wish to master the main points of the theory of electromagnetic and meson processes without having to study the more abstruse parts of field theory". Further: "It is with a view towards establishing closer cooperation [between theoretical and experimental physicists] that this book has been written". In view of the author's long pedagogical experience we may take it for granted that such people exist. The question arises, however, whether it can be the purpose of advanced education in this field to hatch experimental physicists skilled at handling apparatus, but without the deeper fundamental knowledge required for the proper understanding of the problems they are working on. Such perspective must necessarily arouse a feeling of great concern as to the future development of physical research.

One brief remark about the notation. The author designates the charge of a negaton by e and that of a proton by $-e$, thus using the opposite signs to those universally employed. Since the signs are of no consequence in most of the applications, where only the product of the negaton and proton charges appears in the formulae, it is difficult to see any reason for introducing a notation at variance with that internationally adopted.

S. Rozental

EUGENE FEENBERG and GEORGE EDWARD PAKE, *Notes on the quantum theory of angular momentum* (Stanford University Press, Stanford, California, 1953. vii—55 pages. $ 1.25).

This useful booklet, which was reviewed in this journal **1** (1956) **134**, has now been re-issued in a somewhat cheaper form with a paper cover instead of the binding of the previous edition.

L. R.

O.E.E.C., *Application of atomic science in agriculture and food, Vol. II: Present position, future trends and techniques* (European Productivity Agency of the O.E.E.C., Paris, 1959. 247 pages. 21s.).

The first volume devoted by the OEEC to the applications of nuclear physics to agricultural problems was briefly reviewed in this journal **9** (1958/59) **355**. The present volume (reproduced from typescript) surveys, as its title indicates, the present position, future trends and techniques. It is composed of a series of detailed technical reports, preceded by an introduction of synthetic character.

L. R.

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